Lecture 3

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Application of continuum models with microstructure to description of processes of non-mechanical nature at the macro-level: historical remarks and state-of-the-art.

1 Action at a distance and close-range interaction

There exists two concepts of interactions: the action at a distance and the closerange interaction. The concept of the action at a distance consists in the idea that two particles located at some distance interact directly, unassisted some other particles which pass the interaction. The concept of the close-range interaction is based on statement that the particles can interact only when they are in intimate contact with each other. In other words if we observe an interaction of two particles located at some distance then we should conclude that whole space is occupied by a substance and this substance passes the interaction between the particles. Now both the concept of action at a distance and the concept of close-range interaction are used in mechanics. Mechanics of discrete systems (for example, the celestial mechanics, the molecular dynamics, etc.) is principally based on the concept of action at a distance, though the close-range interactions are not ruled out in the discrete mechanics. Continuum mechanics is based on the concept of close-range interaction, though the external loads can be long-range actions. In Middle Ages the most of scientists adhered to the concept of close-range interaction.

2 Philosophy by Descartes

Rene Descartes (1596–1650) was sure that whole space was occupied by substance imperceptible to the touch but capable to act on material bodies and pass the action of one material body on another material body. Descartes called this substance by ether. Descartes was the first who introduced the concept of ether in the science by postulating the mechanical properties of the ether. It is the Descartes's opinion that the particles of the ether permanently move. Since there is no empty space (in Descartes's opinion), moving particles of the ether occupy the places deserted by other particles. In this way the motion of individual particles sets in motion closed circuits of particles, and the motion of these closed circuits forms vortices. Vortices play the important role in the model of Universe originated by Descartes. The main



Figure 1: Rene Descartes

idea of the Descartes's philosophy consists in the statement that we can consider (with a scientific view) the world as an automatic mechanism. Consequently, we can create a mechanical model of any physical phenomenon. Descartes asserted that, analogous to Euclidean geometry, physics could be derived from principles accepted a priory, irrespective of observations and experiments. Most of Descartes's theories was not used in practice. However, his ideas exerted essential influence on subsequent evolution of science.

3 Conception of ether as a fluid substance

Different models of ether were the important part of works of scientists up to beginning of XX century. Starting with Descartes, all scientists of XVII–XVIII centuries conceived the ether as very lightweight (practically weightless) fluid substance which occupies whole "empty" space and all material bodies. Almost all physical processes and phenomena known at that time (except only the contact mechanical interaction and the kinetic theory of heat) were explained on the basis of the concept of ether.







Figure 2: Roger Bacon

Figure 3: Johannes Kepler Fig

Figure 4: Robert Boyle

Starting from antiquity there exist different viewpoints on the heat nature. According to one point of view, the heat is the state of a body. For example, Roger Bacon (1214–1292) and Johannes Kepler (1571–1630) adhered to this opinion. Robert Boyle (1627–1691) believed that the heat is connected with the molecular motion.

Leonhard Euler (1707–1783) supposed that the particles of the combustible material contain the quickly rotating and very elastic matter inside of their envelopes.



Figure 5: Leonhard Euler

If the envelope is destroyed because of some reason then the motion resource which is contained inside of the envelope becomes free.



Figure 6: Galileo Galilei

Galileo Galilei (1564–1642) formulated hypothesis of existence of the imponderable fluid accounting for the heat. This imponderable fluid is dispersed all over the matter and capable to penetrate into bodies. Afterwards this imponderable fluid



Figure 7: Georg Ernst Stahl



Figure 8: Antoine Laurent de Lavoisier

was called the caloric fluid. The caloric fluid was considered to have the following properties. Combining with solids the caloric fluid can transform them into liquids, and combining with liquids the caloric fluid can transform them into gases. In accordance with the theory by Georg Ernst Stahl (1659–1734) all bodies which can burn and oxidize contain the "phlogiston". The "phlogiston" is the combustible substance which moves away from the body during the process of burning and transforms the

body into ashes. Thus, the "phlogiston" in the chemical theory is analogous to the caloric fluid in thermodynamics. Antoine Laurent de Lavoisier (1743–1794), Pierre





Figure 9: Pierre Simon de Laplace Figure 10: Jean Baptiste Joseph Fourier

Simon de Laplace (1749–1827) and Jean Baptiste Joseph Fourier (1768–1830) were adherents of the caloric fluid theory. Success and popularity of the caloric fluid in XVII–XVIII centuries was caused by the fact that predictions of the theory were verified by the experiments carried out at that time.

The caloric fluid theory was recognized to be erroneous only in XIX century when owing to the works by Mayer, Joule, Helmholtz, Kelvin, Clausius and Gibbs, the equivalence principle of heat and energy became firmly established and the heat conservation law which had dominated earlier was completely replaced by the energy balance equation (the first law of thermodynamics). It would seem that the caloric fluid theory should be replaced by the kinetic theory of heat. However, it did not happen in spite of the rapid development of the kinetic theory in XIX–XX centuries. In classical thermodynamics the caloric fluid theory was replaced by the complete absence of whatever interpretation of heat nature. There is no interpretation of heat nature also in the nonequilibrium thermodynamics and continuum mechanics.



Figure 11: Benjamin Franklin



Figure 12: Alessandro Volta

In XVIII century the theory of electricity, as well as the theory of heat, was constructed on the base of the imponderable fluid. It was assumed that the presence of the imponderable fluid in a body causes the body to become electrified. Two models was rival: the single-fluid model and two-fluid model. For example, Benjamin Franklin (1706–1790) and Alessandro Volta (1745–1827) were adherents of the single-fluid model, and Charles Augustin de Coulomb (1736–1806) and Simeon Denis Poisson (1781–1840) were adherents of the two-fluid model. The difference between these models consists in the fact that in the two-fluid model both electric fluids can move relative to the conductor while in the single-fluid model only one electric fluid is mobile. In order to prove advantage of one of the models it was necessary to determine the real motion of electricity. However, it was impossible to do this by experiment. Afterwards it was shown that the both models are equivalent from the mathematical point of view.







Figure 14: Simeon Denis Poisson

In XVIII century the question was actively discussed as to whether the caloric fluid and the electric fluid were different substances or are they the same thing. The following facts were arguments in favor of identity of the electrical and caloric fluids. The electricity and heat can be induced by friction. They both can induce combustion. The electricity and heat can be transferred from one body to another by contact of the bodies. The best conductors of heat are also in general the best conductors of electricity. Two facts were arguments in favor of difference of the electrical and caloric fluids. First, the electrification of a body does not cause any appreciable rise in its temperature. Second, heat penetrates deep into a body, while electricity resides at or near the body surface.



Figure 15: Georg Simon Ohm



Figure 16: Thomas Johann Seebeck

At the beginning of XIX century Georg Simon Ohm (1787–1854) made use of the idea of comparing the flow of electricity in a current to the flow of heat along a wire, the theory of which had been known owing to the publications by Fourier. Ohm supposed that the transition of the electricity from one particle takes place directly only to the one next to it. Ohm believed that the magnitude of the flow between two adjacent particles is proportional to the difference of the "electric forces", just as, in the theory of heat, the heat flow between two particles is considered to be

proportional to the difference of temperatures. Notice that interdependence between the electrical and thermal phenomena was established by experiment only at the beginning of XIX century mainly due to investigations by Thomas Johann Seebeck (1770–1831). In 1822 Seebeck discovered that the electric current in a circuit formed of two different metals can be produced by means of the temperature imbalance.

The concept of electricity evolved in the works of scientists of XVIII century resembles that which nearly a century later was introduced by Faraday. Both concepts explain electrical phenomena without introducing action at a distance. Both concepts suppose that something is present at the spot where any electric action takes place. The difference is that in the theories of XVIII century this something is identified with the electric fluid itself, while in the theories contemporary with Faraday this something is identified with a stress state in the ether. It is interesting to notice that in the interval between the fall of the electric fluid and the rise of the concept by Faraday, the theory of action at a distance was dominant.





Figure 17: Johann Bernoulli

Figure 18: Daniel Bernoulli

Different models of ether was used for explanation of the nature of magnetism. Descartes attempted to explain magnetic phenomena by his theory of vortices. He postulated existence of a vortex of fluid matter round each magnet. This matter enters into the magnet by one pole and leaves the magnet by the other pole, and this matter acts on iron and steel owing to a resistance to its motion afforded by the molecules of those substances. In the middle of XVIII century Leonard Euler, Johann Bernoulli (1667–1748) and Daniel Bernoulli (1700–1782) proposed the explanation of magnetism based on the hypothesis of vertices. The hypothesis of vortices was overthrown by Coulomb who was adherent of the hypothesis of existence of two magnetic fluids (northern and southern). According to Coulomb's assumption the magnetic fluids are permanently connected with the molecules of magnetic bodies so as to be incapable of crossing from one molecule to the next. Each molecule contains equal quantity of the northern and southern fluids, and magnetization consists in a separation of the two fluids to opposite ends of each molecule. Thus it is impossible to separate the two fluids to opposite ends of a body of finite size.

Initially the theory of electricity and the theory of magnetism were developed independently of one another. Although the scientists of XVIII century conjectured that some interconnection between electricity and magnetism can exist, but at that time there were no experimental data verifying this hypothesis. The experimental results were obtained only at the beginning of XIX century. In 1820 Hans Christian





Figure 19: Hans Christian Oersted

Figure 20: Andre Marie Ampere

Oersted (1777–1851) advertised that he discovered the action of electricity on the magnetic needle. Oersted did not determine the quantitative laws of the action, but he only formulated the qualitative effect and gave some remarks on its cause. In Oersted's opinion, the electric perturbation passes through all non-magnetic bodies, while magnetic bodies (or rather their magnetic particles) resist the passage of the electric perturbation. Oersted's viewpoint can be considered as linking the theories of the Cartesian school to the concepts which were introduced subsequently by Faraday. The next step came from Andre Marie Ampere (1775–1836), who in 1825 published the experimental results which demonstrated that two parallel wires carrying currents attract each other if the currents are in the same direction, and repel each other if the currents are in opposite directions.

In XVII–XVIII centuries all variants of explanation of the nature of light were based on the concept of ether. The following question was considered to be controversial one. Is light the oscillations of ether (wave theory of light) or the motion of particles in the ether (corpuscular theory of light)?



Figure 21: Robert Hooke



Figure 22: Christian Huygens

Robert Hooke (1635–1703) suggested a hypothesis that light was the oscillations of ether. In contrast to Descartes (who proposed that a perturbation in the ether was the static), Hooke considered a perturbation in the ether as the quick oscillations of exceedingly small amplitude. Christian Huygens (1629–1695) was adherent of the wave theory of light also. He proposed the following explanation of the variation in velocity of light from one medium to another. The transparent bodies consist of hard particles which interact with the ether, modifying its elasticity. The opacity of metals he explained by supposition that some of the particles of metals should be hard (these account for reflection) and the rest particles should be soft (these account for destruction of penetrating light). Notice that according to the ideas of scientists of XVII–XVIII centuries, the luminiferous ether is fluid, and the light waves propagate in it just as the sound waves propagate in the air. In other words, the light waves were considered to be longitudinal waves. Therefore, in spite of the successful development of the wave theory of light by Huygens this theory was not be able to explain the known at that time experimental facts relating to polarization of light.



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Figure 23: Isaac Newton

Isaac Newton (1642–1727) considered that all space is permeated by an elastic medium or ether, which is capable of propagating the light oscillations. This ether fills up the pores of all material bodies, and is the cause of their molecular bond. The presence of material body causes thickening or rarefaction of the ether. The ether can contain various "ethereal spirits", adapted to produce the phenomena of electricity, magnetism and gravitation. In Newton's opinion, the ether is the intermediary between light and ponderable matter. The light and the ether can interact. It is important to notice that Newton did not consider the light as the ether oscillations. However, Newton considered the heat as the irregular turbulence of the ether. He regarded that the conduction of heat from hot body to cold one is effected by oscillations of the ether, and he supposed that extremely excitation of ethereal motions in burning hot bodies can cause their luminescence.

The idea of existence of the ether was used to explain the nature of gravity. For example, Kepler was sure in existence of only two kinds of interaction between bodies, namely, pressure and shock. From this he concluded that space between the Moon and Earth, as well as the whole space, can not be empty. Huygens explained the nature of gravity by means of the ether. In 1666 Newton discovered the law of gravity. When Newton published it he remarked that his discovery is no concern of the mechanism of gravity. However, this does not mean that he was adherent of the concept of action at a distance. Newton expressed his opinion in clear form having said that to consider that a body can act on another body located at a distance through the vacuum without some "intermediaries" is absurdly, and there are no persons possessing erudition in philosophy which can believe in possibility of this situation. The Newton's law of gravity was regarded with criticality by French Cartesians, and afterwards Newton's progeny rejected Descartes's system of view as a whole including the ether. Although Newton did not formulate a well-defined doctrine on the nature of light, his criticism of the wave theory was misinterpreted by the scientists of the next generation as acceptance of the corpuscular hypothesis. Corpuscles of light were included in the register of chemical elements. At that time particles of the electric and caloric fluids were also included in the register of chemical elements. All these imponderable substances were considered to be chemical elements up to that moment when Lavoisier formulated the fundamental principle that the total weight of chemical agents befor entering into a chemical reaction is equal it after chemical reaction.

The most of scientists of XVII–XVIII centuries did not doubt in existence of substances different from ponderable matter. These substances were considered to be either fluids whose presence provide bodies with some physical properties or media which transmit some interactions between bodies, or the substances acting both stated functions. However, at that time all models of ether were conceived as very lightweight (practically weightless) fluid substances.

It is important to notice that up to the middle of XVIII century all concepts of ether had philosophic and descriptive character. There was no any mathematical models of ether at that time. The first differential equation in the history of science (the equation of equilibrium of torsion fiber) was formulated by Jacob Bernoulli in 1694. The equation of the ideal liquid flow was proposed by Euler in 1755. That is why the construction of mathematical models of ether was impossible up to the middle of XVIII century.

The experimental results accumulated at the end of XVIII century and possibility of mathematical analysis of the ether models resulted in that all known models of ether were recognized to be erroneous. These concepts of ether were rejected. However, there was no ideas what could replace these models. That is why period of domination of the concept of action at a distance began. During this period many interesting and practically important results concerned with the mathematical description of experimentally observed phenomena and solutions of the concrete problems were obtained. Nevertheless, desire to conceive the model explaining the nature of a phenomenon but not only have its mathematical description led to that what in the first half of XIX century the scientists began to elaborate new models of ether based on the principally new ideas.

4 Ether as an elastic solid

Before turn to consideration new type models of ether we discuss the ideas of two scientists of the middle of XVIII century, who approached most close to the concepts which became basic for the scientists of XIX century.

Johann Bernoulli younger (1710–1790) proposed the model of ether which is a fluid, containing a great number of excessively small vortices. The elasticity of the ether appears due to the presence of these whirlpools since owing to centrifugal force, each vortex has a continual tendency to dilate, and so presses against the neighbouring vortices. This aggregate of small vortices (which was called "finegrained turbulent motion" a century and a half later) contains solid corpuscles, whose dimensions are small compared with the distances between them. Vortices push the corpuscles whenever the ether is disturbed, but corpuscles remain near their original positions. A source of light produces perturbation which causes the propagation of oscillations in the ether. Bernoulli compares these oscillations with those of a stretched cord which performs transverse vibrations. Bernoulli's model of ether closely resembles that which was proposed by Maxwell in 1862.



Figure 24: Johann Bernoulli younger

Leonard Euler insisted on the similarity of light and sound. He maintained that light is in the ether the same thing as sound in air. Thus Euler considered light as the longitudinal oscillations. Nevertheless, Euler asserted ahead of Maxwell that the luminous ether was also the source of all electric phenomena. In Euler's opinion, the electricity is a disturbance of ether. A body is electrified when the ether in the pores of the body becomes more or less elastic than the ether in the pores of neighbouring bodies. This occurs when an additional part of ether enters in the body pores or some part of the ether leaves the body pores. In the first case the ether becomes more concentrated and therefore more elastic. In the second case it becomes less concentrated and loses its elasticity. In both cases the equilibrium between the ether in the body and the external ether disturbs and exertion of the ether in the body to return in the equilibrium state causes all electric phenomena. In contrast to most of scientists of XVIII century, Euler identifies electric phenomena with the stress state of ether rather than with the electric fluid itself. In this regard Euler's ideas coincide with view of scientists of XIX century.





Figure 25: Thomas Young

Figure 26: Augustin Jean Fresnel

Thomas Young (1773–1829) and Augustin Jean Fresnel (1788–1827) were the first who supposed that the oscillations of light are performed at right angles to its

direction of propagation. At the same time they pointed out that this peculiarity might be explained by making a new hypothesis regarding the nature of the luminiferous medium. This hypothesis consists in that what the ether behaves analogously to an elastic solid. Fresnel explained the fact that longitudinal waves are absent in the ether by the assumption that the forces of resistance to volume deformations essentially exceeds the forces of resistance to form change, so that the velocity of longitudinal wave propagation essentially exceeds the velocity of light wave propagation and therefore the static equilibrium of pressure takes place.







Figure 28: Claude Louis Navier

At the beginning the theory of ether as an elastic solid met with one obvious difficulty. If the ether has the qualities of a solid, how is it that the planets move through it without encountering any perceptible resistance? This objection was first satisfactorily answered by George Gabriel Stokes (1819–1903), who remarked that such substances as pitch and shoemaker's wax, though so rigid as to be capable of elastic vibration, are yet sufficiently plastic to allow other bodies to pass slowly through them. He suggested that the ether possesses the analogous properties. The Stokes's explanation is an agreement with the Fresnel's hypothesis that the longitudinal wave velocity essentially exceedes the transverse wave velocity, since it is found by experiment with actual substances that the ratio of the velocity of propagation of longitudinal waves to that of transverse waves increases when the medium becomes softer and more plastic.





Figure 29: Augustin Louis Cauchy



Claude Louis Navier (1785–1836) was the first who derived the correct equations of motion of elastic solid of a special kind. Navier started from the hypothesis of central interaction of atoms in a crystal lattice. Therefore his equations contain only one elastic constant. In 1828 Augustin Louis Cauchy (1789–1857) published the paper where the equations of motion of the elastic solid contained two elastic constants were obtained by another method. Afterwards Cauchy extended his theory on the crystal bodies. During ten years Cauchy created two different theories of crystal optics and three different reflection theories on the basis of his equations. All these theories lead to the right or almost right final formulae but contain the incorrect boundary conditions and unreal ratios between the elastic constants. Moreover, the longitudinal oscillations are presented in Cauchy's theories. Cauchy believed that the existence of these oscillations would be verified by experience. He supposed that the longitudinal oscillations have the heat nature. George Green (1793–1841) created two reflection theories. Advantage of these theories over Cauchy's theories consists in the formulations of boundary conditions which is satisfied on the interface of real elastic solids.

The difference between theories constructed on the basis of the analogy with solid consists, first of all, in the formulation of conditions on the interface of two media, but also in other things. In some theories the ether oscillations is considered to occur in parallel with the plane of polarization of light, but in other theories the ether oscillations is assumed to be orthogonal to the plane of polarization. In some theories the inertia is considered to be the same for all media and the stiffness depends on the medium. In other theories, on the contrary, the stiffness is supposed to be the same for all media and the inertia varies according to the medium. There are the ether theories which contain both the transverse and longitudinal waves having propagation velocities of the same order. There are the ether theories in which the velocity of longitudinal waves is much more than the velocity of transverse waves. There exist the theories of unstable (or contractile) ether possessing the zero velocity of longitudinal waves propagation.





Figure 31: William John Macquorn Rankine

Figure 32: John William Strutt

In the theories by Cauchy and Green the right formulae of the anisotropic crystals optics do not come to an agreement with the fact that the oscillations should be orthogonal to the plane of polarization of light. A way of solution of the problem was found by Stokes and afterwards by William John Macquorn Rankine (1820–1872) and John William Strutt, lord Rayleigh (1842–1919). They proposed that the ether in a crystal possesses the various inertia depending on the direction and the isotropic stiffness. The only difficulty is to explain how can inertia be anisotropic. All three authors solved this problem by remarking on the fact that the solid immersed into liquid can have the different effective inertia with respect to different directions. Comparison of the theory based on the hypothesis of anisotropy of inertia with the results of observation did not give satisfactory results. However, afterwards Richard Tetley Glazebrook (1854–1935) shown that integration of Cauchy's theory of unstable ether and the hypothesis of anisotropic inertia gives possibility to obtain the correct formulae of crystal optics.





Figure 33: Richard Tetley Glazebrook

Figure 34: James MacCullagh

James MacCullagh (1809–1847) proposed the original theory, irreproachable in respect of correspondence with the optics experiments. Introduction of a new type of elastic solids is the distinguishing feature of MacCullagh's theory. From the results obtained by Green, MacCullagh concluded that comparing the ether with the customary elastic solid, which resists to compression and form change, it is impossible to explain the optic phenomena satisfactory. As a result, MacCullagh constructed the model of continuum whose internal energy depends on the rotation of volume elements, i.e. on strain measure $\nabla \times \mathbf{u}$ (rotor of the displacement vector). MacCullagh in fact contrived the medium whose oscillations possesses the same properties that the oscillations of light. However, MacCullagh's theory gave rise to doubts of both contemporaries and the scientists of next generation. This theory was appreciated only in 40 years later when FitzGerald attracted attention to it.

In the middle of XIX century the scientists paid attention to the optic properties of metals. The opacity and capacity to reflect light for all hades are characteristic properties of metals. Cauchy and MacCullagh proposed the mathematical description of the medium possessing such properties. The equation of motion of this medium differs from the equation of motion of the elastic solid by the term containing the first time derivative. This term characterizes the viscous properties of the medium owing to which the dissipation of the energy of light oscillations occurs and the medium turns out opaque. The term causing opacity causes the features of metal reflection. Notice that this model is in a good agreement with the idea by Huygens that metals are characterized by the presence of soft particles which damp the light oscillations.

The representation of ether properties by means of the properties of solid is important part of works of the scientists of XIX century. Interest in it has relaxed only after creation of the electromagnetic theory of light. Although the models of ether as the solid have not been applied scientists created these models have obtained many interesting and practically important results in both the theory differential equations and mechanics of solids. Thus, attempts to construct the models of ether contribute to development of other fields of science.





Figure 35: Joseph Valentin Boussinesq

Figure 36: Franz Ernst Neumann

The theory of ether by Joseph Valentin Boussinesq (1842–1929) deserves the especial discussion. Boussinesq's theory essentially differs from all theories considered above. According to this theory, all material bodies as well as interplanetary space are occupied by exactly the same ether in regard both to inertia and to rigidity. Boussinesq assumed that all ether processes should be represented by two systems of equation. The first system of equations are equation of the ether motion. The second system of equations should describe the interaction of the ether and matter. In Boussinesq's opinion, the optical properties of matter are due to interaction between the ether and the material particles. Franz Ernst Neumann (1798–1895) stated the analogous ideas. In many years later these ideas were revived in connection with electromagnetic theory.

5 Models of ether and electromagnetism

At the beginning of XIX century interplay of the electricity and magnetism was already known. However, identity of the electricity obtained by friction and the electricity obtained from galvanic battery was opened to question. Michael Faraday (1791–1867) has proved that the process in the conductor connecting outlets of a galvanic cell has the same nature as the process proceeding in the conductor by which a condenser discharges in a short space of time. Faraday, in contrast to his predecessors and most of progeny, refrained from asserting that this process consists in the real motion of some substance. To explain phenomena relating to the different areas of physics Faraday uses the concept of the action extending in a medium gradually due to the influence of contiguous particles. He explains the electric conduction by the contiguous particles action which depends on the forces arising in the process of electric excitative. These forces bring the particles into the stressed state (polarization state). Being in the stressed state the contiguous particles act on each other. As a result, the stress relaxes and consequently discharge occurs. Faraday believed that the difference between insulators and conductors in the fact that particles of insulators can remain in the polarization state and particles of conductors can not be permanently polarized. Faraday assumed that the action of the magnetic field can be passed at a distance by means of intermediate particles by the same way as the action of static electricity can be passed at a distance. The mechanism of action transfer consists in that the intermediate particles temporarily get into the same state as particles of the transmitting source.



Figure 37: Michael Faraday

Scientists had been long illustrated magnetism by strewing iron filings on a sheet of paper, and observing the curves in which they dispose themselves when a magnet is brought under the sheet. These curves suggested to Faraday the idea of lines of magnetic force, or curves whose direction at every point coincides with the direction of the magnetic intensity at that point. Afterwards by analogy with the lines of magnetic force, Faraday introduced lines of electric force, or curves whose direction at every point coincides with the direction of the electric intensity at that point. Faraday believed that whole space is occupied by lines of electric and magnetic force and he considered light and infrared radiation as the transverse oscillations which propagate along the force lines. Faraday's conception was presage of appearance of the electromagnetic theory of light.

Although Faraday was not adherent of idea of action at a distance, he can not be considered as adherent of the ether theory in the strict sense. Faraday originated his own research method. All arguments by Faraday are based on the force lines which are considered as the physical reality rather than a mathematical abstraction. In fact, in Faraday's argumentation the force lines are the analogue to the ether in the sense that both are assumed to be some physical reality different from the ponderable matter. Notice that Faraday used the method of argumentation different from that what was adopted in that time. Therefore many of the contemporaries of Faraday hardly conceived his method.





Figure 38: Gustav Theodor Fechner

Figure 39: Wilhelm Eduard Weber

In contrast to Faraday, the most of scientists imagined the electric current as a flow of some substance. Gustav Theodor Fechner (1801–1887) supposed that the

electric current represents the motion of electric charges. Wilhelm Eduard Weber (1804–1890), Fechner's progeny, is the author of the first electronic theory, i.e. the theory which explains electrodynamic phenomena by the activity of moving electric charges under the action of forces depending on both their positions (as in electro-static) and their velocities.





Figure 40: Carolus Fridericus Gauss Figure 41: Georg Friedrich Bernhard Riemann

Carolus Fridericus Gauss (1777–1855) proposed to himself to supplement the known forces which act between electric charges by other forces, such as would cause electric actions to propagate between the charges with a finite velocity. Gauss decided not to publish his researches until he should have devised a mechanism by which the transmission could be clear to be effected. However he had not succeeded in solving this problem. Georg Friedrich Bernhard Riemann (1826–1866), Gauss's pupil, attempted to realize Gauss's aspiration. Riemann proposed a model of ether whose elementary volume can resist to compression and also can resist to changes of orientation (like the elementary volume of MacCullagh's ether). The former property Riemann conceived to be the cause of gravitational and electrostatic effects, and the latter to be the cause of optical and magnetic phenomena.



Figure 42: William Thomson (Lord Kelvin)

William Thomson, Lord Kelvin (1824–1907) was successful to a greater extent than other scientists in creation of the mechanical models of physical phenomena and invention of the analogues between different physical processes. In one of his earliest papers (written when he was a the first year student of Cambridge) Thomson compared the expansion of electrostatic force in a region containing electrified conductors with the expansion of the heat flow in an infinite solid. The equipotential surfaces in the one case correspond to the isothermal surfaces in the other case, and an electric charge corresponds to a heat source. This work by Thomson is interesting due to Thomson proved that formulae which relate to the electric field (and which before deduced from laws of action at a distance) are identical with formulae relating to the heat theory which have been deduced from hypotheses of action between contiguous particles. Thus Thomson was the first who introduced in science at the level of mathematical description the idea of electric action which is transmitted by means of a continual medium.

Afterwards (1846) Thomson found out the analogies of electric phenomena with those of elasticity. He analyzed the equations of equilibrium of an incompressible elastic solid and showed that the distribution of the displacement can be identified with the distribution of the electric force in an electrostatic system. However, Thomson did not restrict himself to this analogy and proposed another one. He demonstrated that the rotor of elastic displacement $\nabla \times \mathbf{u}$ can be identified with the vector of magnetic induction.



Figure 43: James Clerk Maxwell

In a ten year later (1856) James Clerk Maxwell (1831–1879) informed Cambridge philosophical society on his first attempt to create the mechanical concept of electromagnetic field. He considered the illustration of Faraday's lines of force, studied their properties and shown that the vector of magnetic induction can be identified with the velocity vector of the incompressible liquid. Some years previously such an analogy was indicated by Faraday, who assumed that along the lines of magnetic force there exists a "dynamic station" analogous to that of the electric current, and that the physical lines of magnetic force are electric currents in fact. Notice that the comparison with the lines of a liquid flow is applicable both to magnetic and electric lines of force. However, in comparing of the velocity vector of the incompressible liquid with electric induction it is necessary to introduce into the liquid the sources and sinks corresponding to the electric charges. Thus, the magnetic analogy is simpler. In the latter half of his work fulfilled in 1856 Maxwell attempted to connect the ideas of Faraday with the mathematical analogies by Thomson which was the identification of magnetic induction with $\nabla \times \mathbf{u}$. As a result of this research Maxwell determined, in particular, the relation between current strength and magnetic field strength vector.

In the same year (1856) Thomson proposed an alternative interpretation of magnetism. From a study of the rotation of the plane of polarization of light under the action of magnet Thomson concluded that magnetism possesses a rotatory character. He assumed that the resultant angular momentum of the thermal motions of a body can be interpreted as the measure of the magnetic moment. Thomson believed that the explanation of all phenomena of electromagnetic attraction or repulsion as well as electromagnetic induction should be looked for in the inertia or pressure of the matter of which the motions constitute heat. He did not express an definite opinion relative to what is the nature of the matter and whether this matter is or is not electricity. Moreover, he was sure that under current state of science there was no sense to ponder over this problem.

Thomson's arguments convinced Maxwell in the fact that magnetism is the phenomenon of rotational character, and currents are the phenomena of translational character. Nevertheless, the alternative analogy, according to which the electric phenomena is considered to be rotational and magnetic phenomena is supposed to be translational, was a long time attracting attention of scientists. For example, Hermann Ludwig Ferdinand von Helmholtz (1821–1894) and Gustav Robert Kirchhoff (1824–1887) were among the scientists who developed this concept.





Figure 44: Hermann von Helmholtz Figure 45: Gustav Robert Kirchhoff

George Francis FitzGerald (1851–1901) used an analogy with the model of elastic solid which had been proposed by MacCullagh. Remind that only this model permits the propagation of waves possessing the properties of light waves. With accordance with the analogy by FitzGerald, electric displacement corresponds to the twist of the elementary volume of ether, and electric charge should be represented as an intrinsic rotational strain. Thus, FitzGerald proposed the model which describes not only optic phenomena but also electric and magnetic interactions.



Figure 46: George Francis FitzGerald

None of the discussed attempts to represent electric and magnetic phenomena by means of motions and stresses of a continuum is devoid of imperfections which were evident, first of all, to the authors of these models. That is why subsequent evolution of the models of ether consisted in their complication.

The Maxwell's model proposed in 1862 is based on the concept of magnetism as the phenomenon of rotational character. In accordance with Faraday's ideas, Maxwell supposed that the ether is a medium rotating about the lines of magnetic force, and each unit tube of force can be presented as an isolated vortex. There is an evident problem in this model. Since two neighbouring vortices rotate in the same direction, the particles in the circumference of one vortex must be moving in the opposite direction to the particles contiguous to them in the circumference of the contiguous vortex. Therefore the motion is discontinuous. In order to escape from this difficulty Maxwell used a simple technique. When two wheels should revolve in the same direction, an "idle" wheel is inserted between them (see Fig. 47). In fact, this model is a two-component medium. In this model "magnetic medium" is divided into cells by the walls consisting of separated layer of spherical particles which are "electricity". The cell substance is elastic both with respect to compression and with respect to form change. The connection between the cells and the particles forming walls is that rolling without sliding and the tangential action on each other take place. When sells rotating the stress state equivalent to combination of the hydrostatic pressure and the longitudinal stress along the rotation axes arises.



Figure 47: Maxwell's model

On the basis of his model Maxwell proposed the mathematical description of electrodynamics in the form of system of equation which is called by his name. A characteristic feature of Maxwell's theory is the fact that magnetic energy is the kinetic energy and that electric energy is the internal energy. This conception, for which Maxwell was indebted to Faraday and Thomson, brought together the electromagnetic theory and the theories of ether as the elastic solid. Creation of an electromagnetic theory of light was the logical result of that. By that time it had been determined by experiment that the value of constant in Maxwell's equations is identical to the velocity of light. This result was very important since it allowed Maxwell to maintain that the light consists in the transverse wave motion of the same medium which causes electric and magnetic phenomena.

In 1885 FitzGerald proposed a model resembling the Maxwell's model. This model is based on the mechanism constituted of a number of wheels, free to rotate on axes fixed perpendicularly in a plane board (see Fig. 48). The axes are fixed at the intersections of two systems of perpendicular lines, and each wheel is connected



Figure 48: FitzGerald's model

to each of its four neighbours by an rubber band. If all wheels are rotating with the same velocity then there is no stresses in the system. If some of the wheels are rotating faster than others, the rubber bands are strained. It is evident that the wheels in this model play the same role as the vortices in Maxwell's model. A strain on the bands represents dielectric polarization. Conduction is represented by a slipping of the bands on the wheels.

The reason which prevent from accepting the rotationally elastic ether arose mainly from the want of any clear example of a body possessing such a property. This difficulty was removed in 1889 by Thomson (Lord Kelvin), who designed several mechanical models possessing rotational elasticity. In contrast to the models by Maxwell and FitzGerald, the models by Kelvin were three-dimensional ones.



Figure 49: Kelvin's continuum

As a result of his research, Kelvin created the model of continuum consisting of the axially symmetrical particles (see Fig. 49) which can freely rotate about their exes, whereas the rest displacements and rotations meet with resistance. It was the first model in the history of science, such that the displacements and rotations were introduced as independent degrees of freedom. Notice that the model of medium with rotational degrees of freedom was created by Kelvin in the form of technical description. The possibility of construction of the mathematical models of such media came many years later, when in 1909 brother E. and F. Cosserat developed the method of description of 3D-continuum with rotational degrees of freedom.

6 Theory of ether and the nature of ponderable matter

Thomson (Lord Kelvin) was the first who attempted to originate a general physical theory based on the vortex motion. He assumed that the atoms of matter consist of vortex rings in the ideal liquid. To explain the interaction of atoms Thomson used the analogy with the behaviour of smoke rings, and he explained the spectroscopic properties of matter by the fact that vortex rings have natural periods.

Having considered the nature of ponderable matter, Faraday supposed that an elementary atom is the force field (electric, magnetic and gravitational) surrounding a center point. In accordance with Faraday's hypothesis the atom does not have the certain size. It extends through the whole space and is absolutely permeable. Correspondingly, molecules consist of the force spheres that penetrate into each other rather than of the atoms located near to each other. According to the concept by Faraday the ether completely specifies the structure of ponderable matter rather than only provides the ponderable matter with the certain properties as it was assumed by other scientists. It can be argued that the essence of Faraday's concept consists in that the ponderable matter is the certain state of ether.

Maxwell, as well as Faraday and Thomson, considered the ponderable matter as the modification of ether which differs from the ether occupying the "empty" space by the values of permittivity and permeability only.

7 The main tendency of development of the ether theory and outlooks

Throughout the history of ether theory many scientists have proposed the models of different physical phenomena and processes which are based on the conception of rotational character of motion of ether. Initially there were the conception of vortex motion of fluid (Descartes's philosophy) and the conception of rotation of elementary volume of solid (MacCullagh's ether), i.e. conceptions based on the models of continua possessing the translational degrees of freedom only. In the second half of XIX century the models in which the rotational motions are introduced as independent degrees of freedom (models by Maxwell, FitzGerald and Kelvin) have been proposed. The models of ether based on the translational degrees of freedom can not explain all facts known from experiments.

The models of ether based on the rotational degrees of freedom, unfortunately, have not been developed. The reason is the fact that in the second half of XIX century the level of development of continuum mechanics made it impossible to describe 3D-continua with rotational degrees of freedom, and in the first decade of XX century scientists have almost completely refused the concept of the ether. The reason for this is that all attempts to observe the Earth's motion relative to ether were unsuccessful. As a result the interplanetary space was seen as empty and not having any properties but nevertheless able to propagate electromagnetic waves.

The development of quantum physics led to the empty space (vacuum) is provided by some properties similar to those of the matter. For example, the vacuum is considered to have a temperature different from absolute zero; the vacuum polarization is possible; and there exists such conception as a zero-point energy of vacuum. In preface to the book "A history of the theories of ether and electricity" E. Whittaker writes that it seems absurd to retain the name of the "vacuum" for the category having so many physical properties and that the term "ether" is quite appropriate. However, the question is not only the use of a term. The essence of the matter is that every point in space is provided with certain properties, which are characterized by a set of constants, and at every point in space there are certain processes that are described by differential equations. Consequently, these processes can be represented as processes existent in a continuous medium.



Figure 50: Pavel Andreevich Zhilin

For a long time there was no followers of the Cosserat's approach to construction of mathematical models of continua with the rotational degrees of freedom. However, starting with the works by C. Truesdell and J. Ericksen, written in the second half of XX century, this approach began to intensively develop. Now this approach is quite well developed and the derivation of basic equations is not difficult. Thus, at the close of XX century the possibility of mathematical realization of the ideas analogous to ideas by Maxwell, FitzGerald and Kelvin appears. At the turn of XX– XXI centuries Pavel Andreevich Zhilin (1942–2005) created the mechanical models of physical processes. He proposed the model of electromagnetic field that is based only on the rotational degrees of freedom and the model of the elastic continuum (also based only on the rotational degrees of freedom) whose mathematical description can be reduced to the Schrödinger equation and the Klein–Gordon equation.