Methods of Continuum Mechanics in the Problems of Nanotechnology

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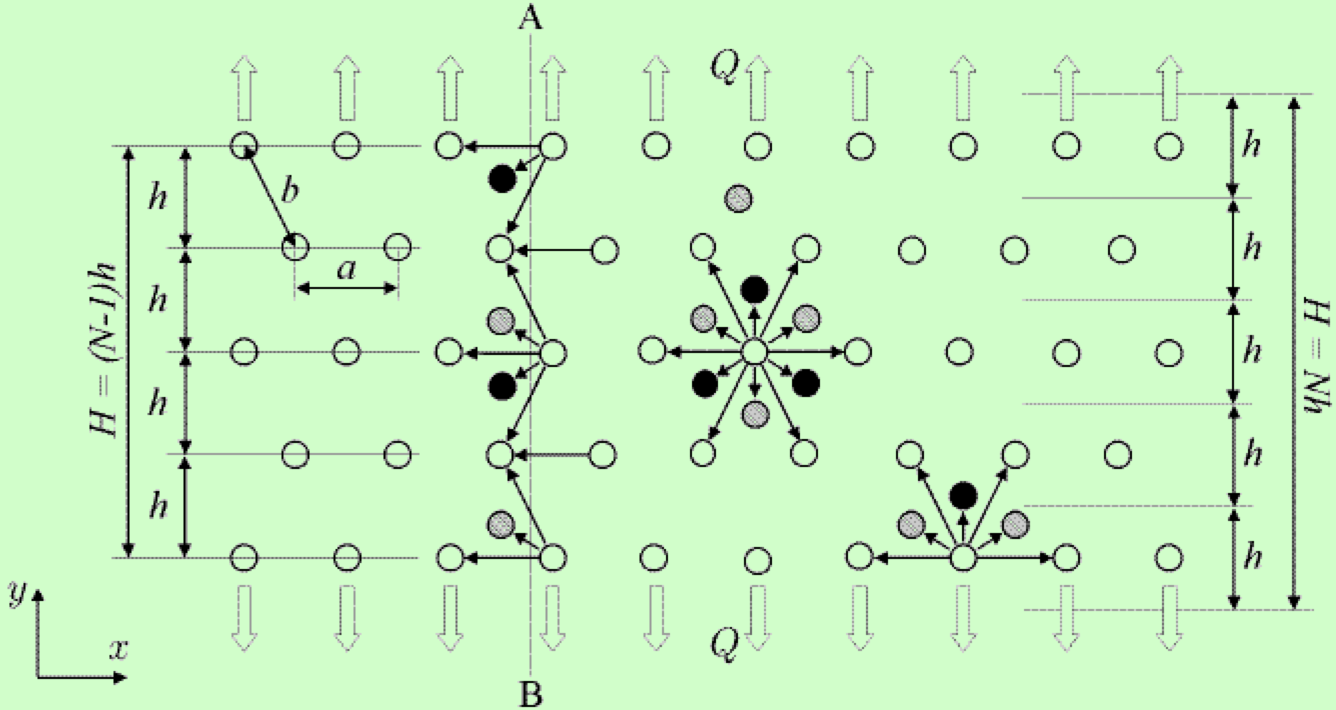
Anomalism in mechanical properties of nanosize objects



Model of finite 2D (3D) crystal.

White circles represent 2D crystal or medium layer of 3D crystal.

Gray / black circles represent atoms from upper / lower layers (3D).

The calculated values for the moduli (2D)

$$\nu_1 = \nu_\infty, \qquad E_1 = \frac{N}{N_*} E_\infty;$$

$$\nu_2 = \frac{N-1}{N-\frac{1}{9}} \nu_\infty, \qquad E_2 = \frac{N}{N-\frac{1}{9}} E_\infty.$$

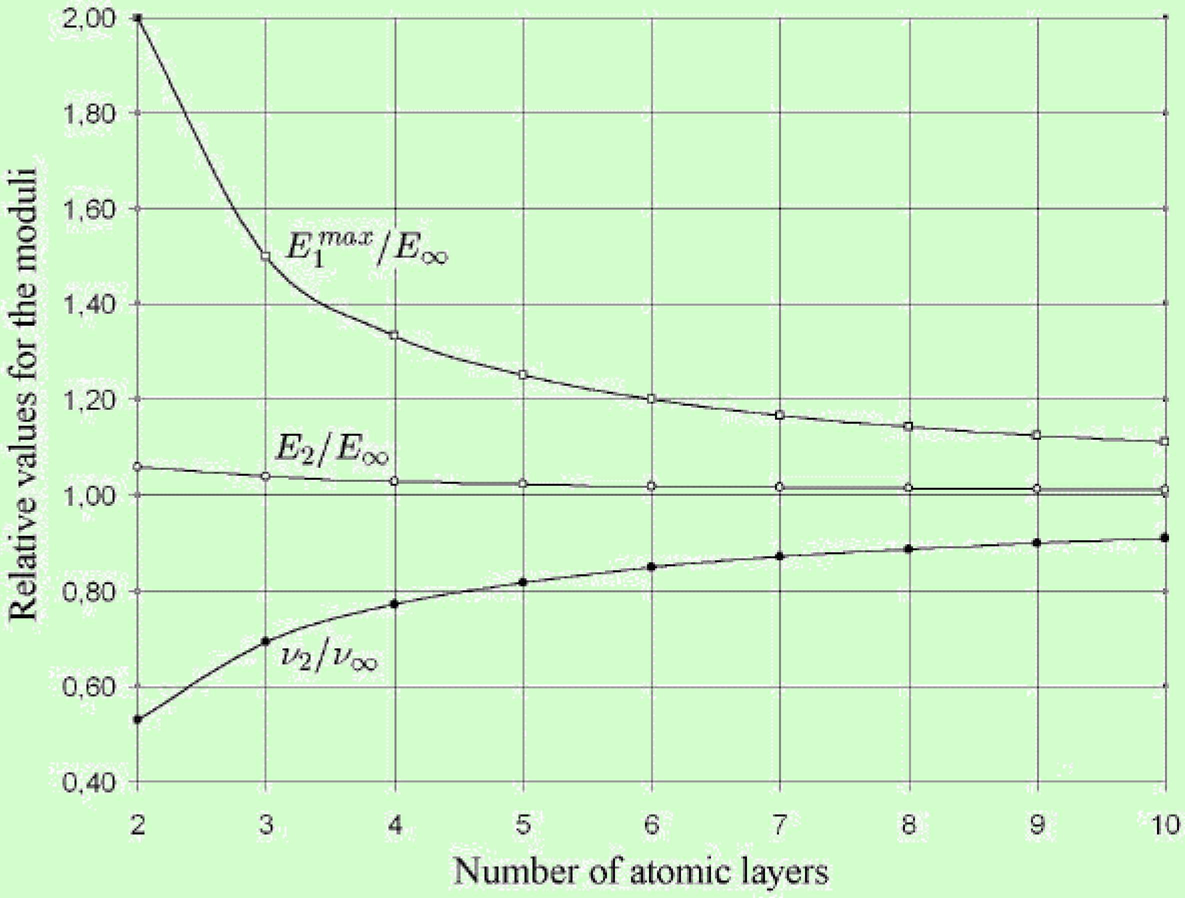
ν — Poisson coefficient, E — Young modulus;

N — number of atomic layers in x (2D) and z (3D) directions;

N_* — an ambiguous parameter ($N-1 \le N_* \le N$).

Indexes 1 and 2 correspond to tension in x and y directions.

Index ∞ corresponds to macroscopic case ($N \to \infty$).



Dependence of the Young modulus and Poisson ratio on the number of atomic layers (2D).

Dimension	Parameter	Designation	Min value	Max value
2D	Young modulus	E/E_∞	1.00	2.00
3D	Young modulus	E/E_∞	0.87	3.83
2D	Poisson coefficient	ν/ν_∞	0.53	1.00
3D	Poisson coefficient	ν/ν_∞	0.57	1.12

Maximum deviations from macroscopic values, 2-layer nanocrystals.
(3D calculations by O. S. Loboda.)

Conclusions

- Deviation of mechanical properties from macroscopic values is proportional to $1/N$, where N is the number of atomic layers.
- The values of mechanical properties essentially depends on the definition of material volume, which is ambiguous at nanoscale.
- Nanoscale effects are more propaunced in 3D then in 2D.

Bending stiffness of nanocrystals

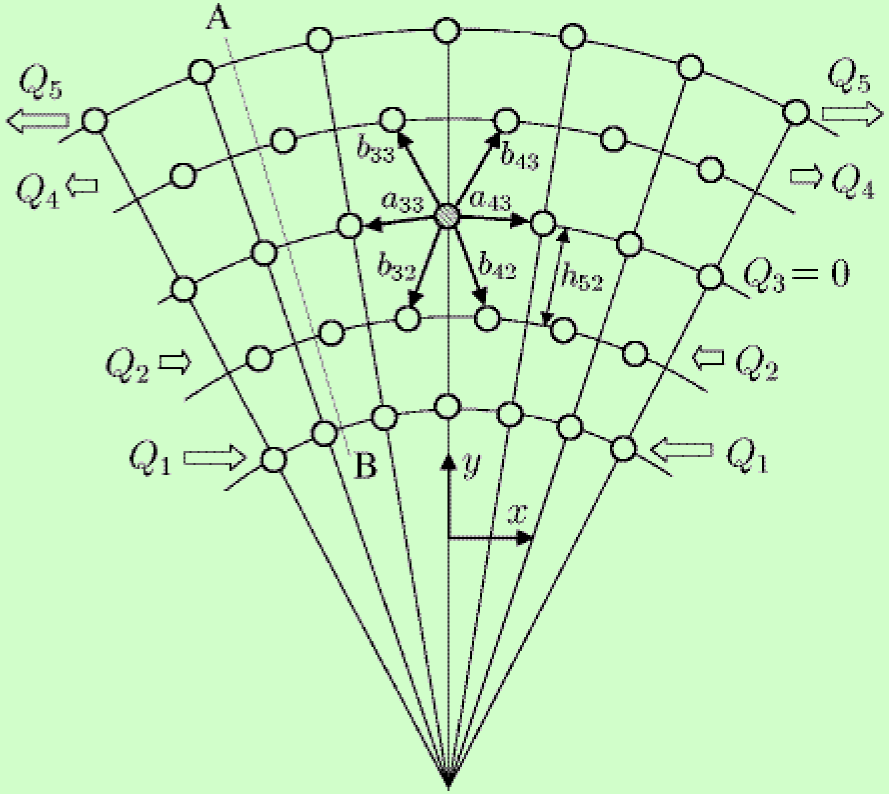


Figure 1: Bending of nanocrystal strip.

Formulation of problem:

$N \geq 2$ layers in y direction, $J >> N$ layers in x direction.

From one layer to another forces Q_n vary linearly, so that

$$\sum_{n=1}^{N} Q_n = 0, \qquad \sum_{n=1}^{N} R_n Q_n = M.$$

Interaction between atoms:

$$F(a_{jn}) = C\Delta a_{jn}, \qquad F(b_{jn}) = C\Delta b_{jn}, \qquad C \stackrel{\text{def}}{=} F'(a_0) > 0.$$

Solution of equations of equilibrium for atoms of strip:

$$\Delta b_{jn} = 0, \qquad \Delta a_{jn} = \frac{Q_n}{C}.$$

Angle between heighboring atomic layers α and curvature æ:

$$\alpha \stackrel{\text{def}}{=} \frac{\Delta a_{jN}/2 - \Delta a_{j1}/2}{h_0(N-1)}, \qquad æ \stackrel{\text{def}}{=} \frac{\alpha}{a_0/2}.$$

Bending stiffness of monocrystal strip:

$$D \overset{\text{def}}{=} \frac{M}{\beta} = \frac{Ca_0^3}{16}(N-1)N(N+1).$$

Bending stiffness in terms of macroscopic parameters:

$$D = \frac{E_1 H^3 (N^2-1)}{12 N_*^2}.$$

Let be $N_* = N$. Then $E_1 = E_\infty$. In this case:

$$D = D_\infty \left(1 - \frac{1}{N^2}\right), \qquad D_\infty = \frac{E_\infty H^3}{12}, \qquad H = N h_0.$$

Here D_∞ is bending stiffness of strip, known from microscopic theory.

Dependence of parameter $k = D/D_\infty$ from number of atomic layers:

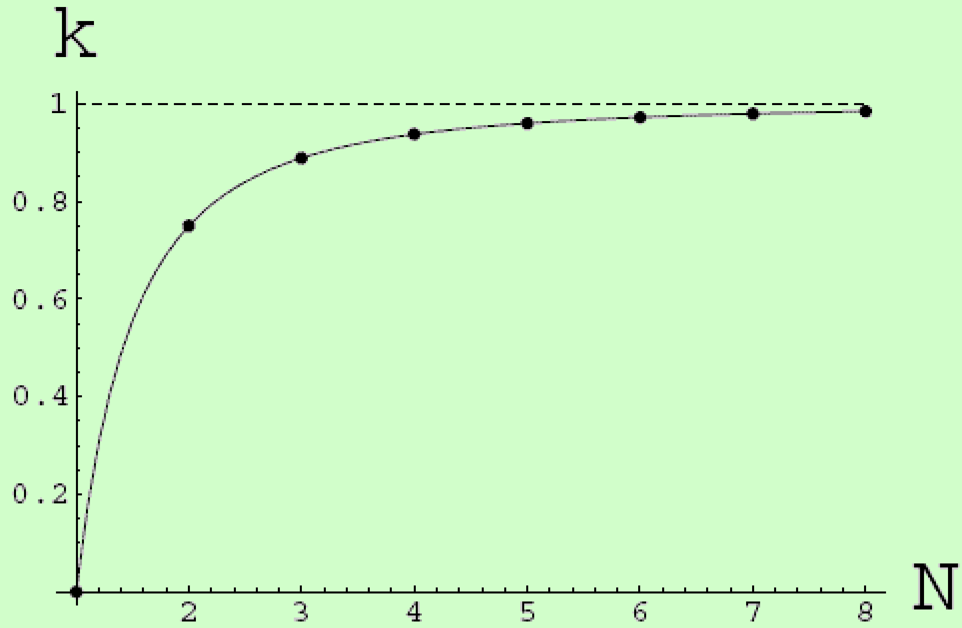


Figure 2: Dependence of bending stiffness from number of layers.

Problem: The single atomic layer forming the nanotube would have zero bending stiffness, so that such nanotube would be unstable.

Mechanical models of atom

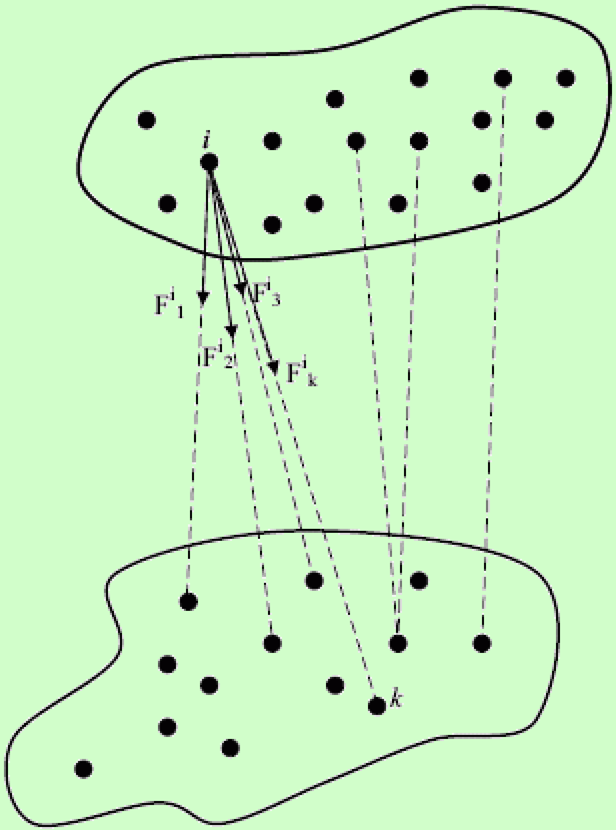


Figure 3: Interaction of particles of general kind.

$$\underline{F} = \sum_{i,k} \underline{F}^i_k, \qquad \underline{M}^Q = \sum_{i,k} (\underline{r}_i - \underline{r}_Q) \times \underline{F}^i_k.$$

Different mechanical models of atom

- Atom is considered as a material point. Microstructure of atom is ignored. Atom has translation degrees of freedom only. Atoms interact by the central forces.
- Atom is considered as a body-point. Microstructure of atom is ignored. Atom has translation and rotation degrees of freedom. Atoms interact by the forces and moments. Forces are not central.
- Atom is considered as a particle with microstructure. Atom has internal degrees of freedom.

Bending stiffness of nanocrystals

Rotational degrees of freedom and moment interactions of atoms are taken into account.

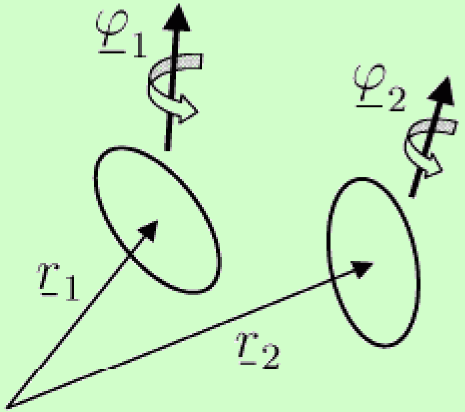


Figure 4: Moment interaction of two atoms.

Interaction between two atoms:

$$\underline{F} = \underline{\underline{A}} \cdot \underline{\varepsilon}, \qquad \underline{M} = \underline{\underline{C}} \cdot \underline{\kappa},$$

$$\underline{\varepsilon} = \underline{r} - \underline{r}_0 + \frac{1}{2}\underline{r}_0 \times (\underline{\varphi}_1 + \underline{\varphi}_2), \quad \underline{\kappa} = \underline{\varphi}_2 - \underline{\varphi}_1, \quad \underline{r} = \underline{r}_2 - \underline{r}_1,$$

$$\underline{\underline{A}} = C_1 \frac{\underline{r}_0 \underline{r}_0}{|\underline{r}_0|^2} + C_1^* \frac{\underline{k} \times \underline{r}_0 \, \underline{k} \times \underline{r}_0}{|\underline{r}_0|^2}, \qquad \underline{\underline{C}} = C_2 \underline{k}\,\underline{k}.$$

Here $\underline{F}$ — force vector, $\underline{M}$ — moment vector,

$\underline{\varepsilon}$, $\underline{\kappa}$ — strain vectors, $\underline{\underline{A}}$, $\underline{\underline{C}}$ — stiffness tensors.

$\underline{k}$ — unit vector, perpendicular to the strip plane,

in the equilibrium position $\underline{r} = \underline{r}_0$.

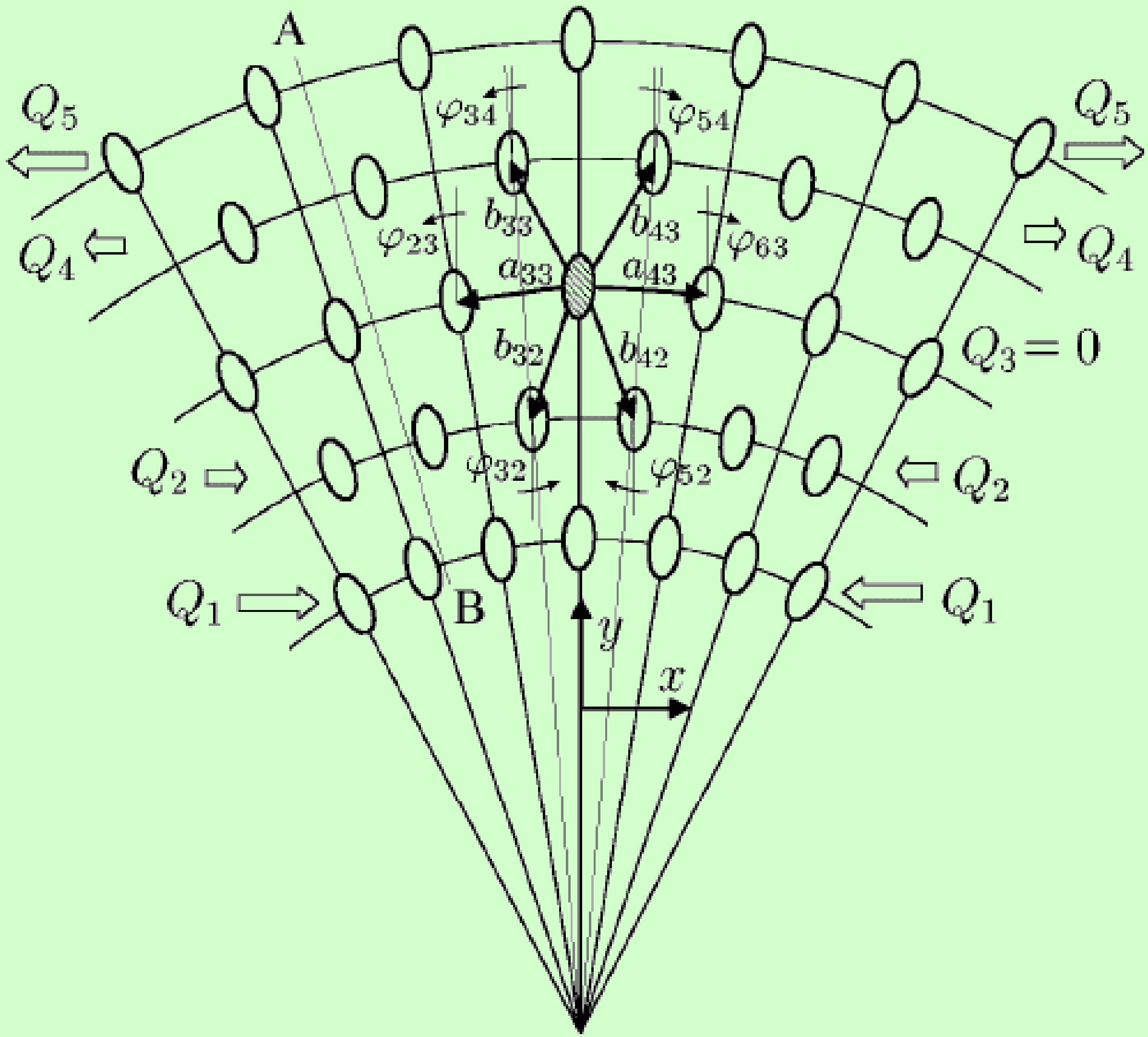


Figure 5: Bending of nanocrystal strip.

Formulation of problem:

From one layer to another forces Q_n vary linearly, so that

$$\sum_{n=1}^{N} Q_n = 0, \qquad \sum_{n=1}^{N} R_n Q_n = M_\Sigma.$$

The crystal sides rotate as a rigid body.

Bending stiffness:

$$D = \frac{C_1 a_0^3}{16}(N-1)N(N+1) + \frac{C_2 a_0}{2}(3N-1).$$

Bending stiffness in terms of macroscopic parameters:

$$D = D_\infty \left(1 - \frac{1}{N^2}\right) + \tilde{E}_\infty H \left(1 - \frac{1}{3N}\right), \qquad H = N h_0.$$

Here $\tilde{E}_\infty$ is elastic modulus, characterizing rotational stiffness, known from the moment microscopic theory of elasticity.

Method of experimental determination of bending stiffness of nano-objects

Idea of experiment.

- Let us consider two similar objects:
  one of them is nano-size object; another is macro-size object.
- Let us find eigenfrequencies of considered objects $\omega_n^{(1)}$ and $\omega_n^{(2)}$ by the experimental method.
- We can select geometrical parameters of considered objects and boundary conditions so that $\dfrac{\omega_n^{(1)}}{\omega_n^{(2)}} = f(D_1, D_2) = \text{const}.$
- If we know $\omega_n^{(1)}$, $\omega_n^{(2)}$ and bending stiffness of macro-object D_2, we can calculate bending stiffness of nano-object D_1.

1. **Cantilever** (elastic rod)

Boundary conditions:

$$w(0) = 0, \qquad w'(0) = 0, \qquad w''(l) = 0, \qquad w'''(l) = 0.$$

Eigenfrequencies are depend on following parameters:

$$\omega_n = \sqrt{\frac{D}{\rho l^4}}\,\Omega_n, \qquad \text{where } \Omega_n \text{ depends on } n \text{ only.}$$

Let us consider two rods, having different physical and geometrical characteristics. Then

$$\forall\, n: \qquad \Omega_n^{(1)} = \Omega_n^{(2)}. \qquad \Rightarrow \qquad \frac{\omega_n^{(1)}}{\omega_n^{(2)}} = \sqrt{\frac{D_1 \rho_2 l_2^4}{D_2 \rho_1 l_1^4}}.$$

2. Rectangular plate

Dimension of plate: $-a \le x \le a, \quad -b \le y \le b.$

Boundary conditions:

$$w|_{x=\pm a} = 0, \quad \frac{\partial^2 w}{\partial x^2}\Big|_{x=\pm a} = 0, \quad w|_{y=\pm b} = 0, \quad \frac{\partial^2 w}{\partial y^2}\Big|_{y=\pm b} = 0.$$

Eigenfrequencies are depend on following parameters:

$$\omega_{nm} = \sqrt{\frac{D}{\rho a^4}}\,\Omega_{nm}, \qquad \Omega_{nm} = \left(m - \frac{1}{2}\right)^2 \pi^2 + \left(n - \frac{1}{2}\right)^2 \pi^2 \left(\frac{a}{b}\right)^2.$$

Let us consider two plates, having different physical and geometrical characteristics, but identical parameter a/b. Then

$$\forall\, n,\, m: \qquad \Omega_{nm}^{(1)} = \Omega_{nm}^{(2)}. \qquad \Rightarrow \qquad \frac{\omega_{nm}^{(1)}}{\omega_{nm}^{(21)}} = \sqrt{\frac{D_1 \rho_2 a_2^4}{D_2 \rho_1 a_1^4}}.$$

3. Cylindrical shell (bending deformations only)

Dimension of cylindrical shell: R — radius of cylinder.
Cylindrical coordinates: z, θ.

Boundary conditions:

$$u_\theta(0,t) = 0, \qquad w(0,t) = 0, \qquad \varphi_z(0,t) = 0,$$

$$u_\theta(\pi,t) = u_\theta(-\pi,t), \quad w(\pi,t) = w(-\pi,t), \quad \varphi_z(\pi,t) = \varphi_z(-\pi,t).$$

Eigenfrequencies are depend on following parameters:

$$\omega_n = \sqrt{\frac{D}{\rho R^4}}\,\Omega_n, \qquad \text{where } \Omega_n \text{ depends on } n \text{ only.}$$

Let us consider two shells, having different physical and geometrical characteristics. Then

$$\forall\, n: \qquad \Omega_n^{(1)} = \Omega_n^{(2)} \qquad \Rightarrow \qquad \frac{\omega_n^{(1)}}{\omega_n^{(2)}} = \sqrt{\frac{D_1 \rho_2 R_2^4}{D_2 \rho_1 R_1^4}}.$$

4. **Cylindrical spiral shell** (bending deformations only)

Experiments by V. Prinz (Novosibirsk)

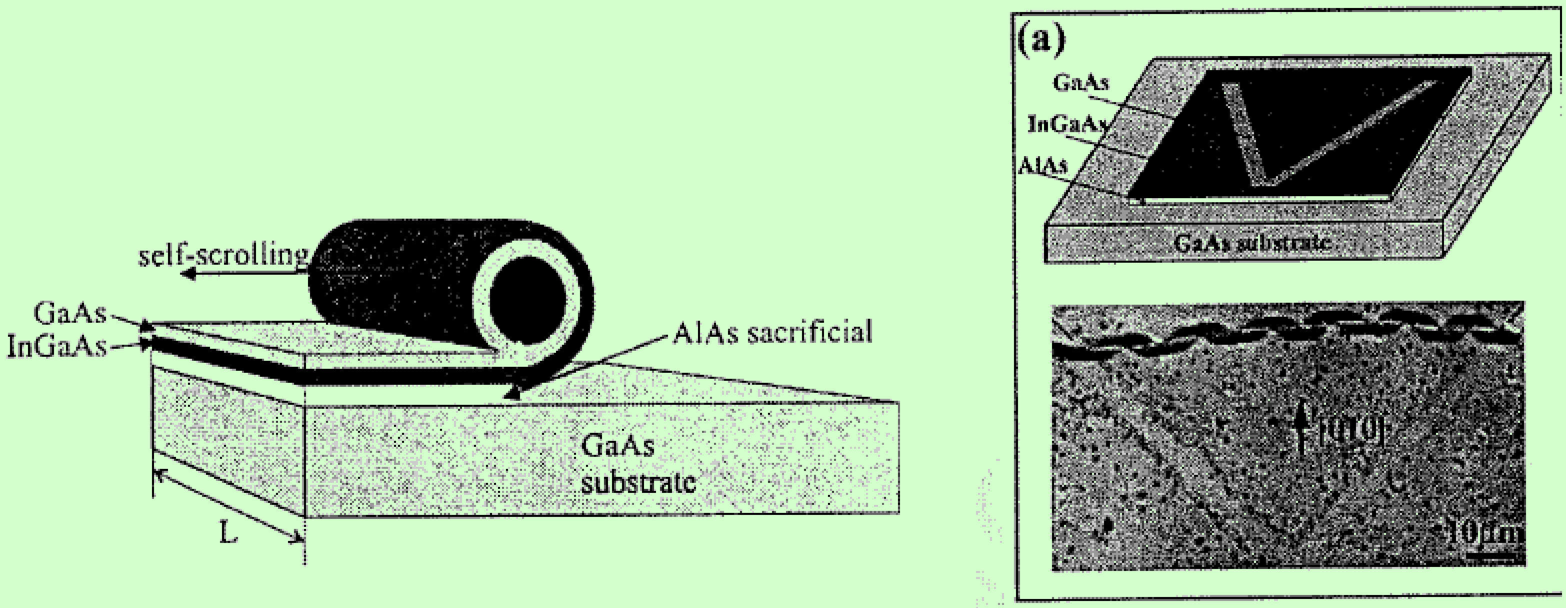


Figure 6: Cylindrical shell and cylindrical spiral shell.

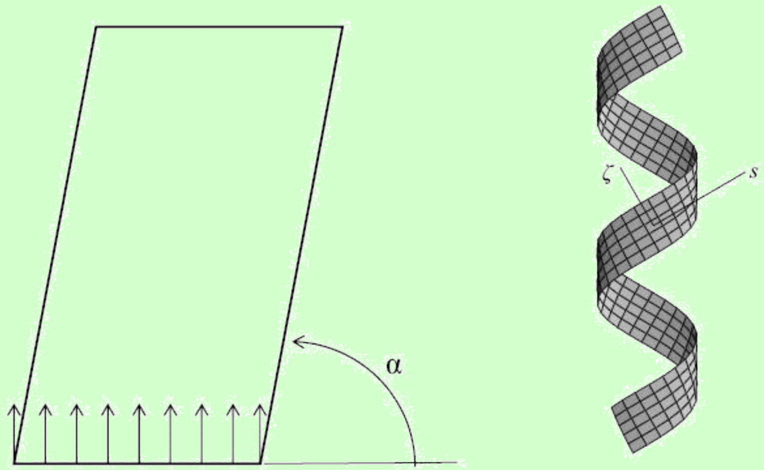


Figure 7: Cylindrical spiral shell.

Geometry of cylindrical spiral shell.

Spiral coordinates s, ζ and cylindrical coordinates z, φ:

$$z = R(\sin\alpha\, s + \cos\alpha\, \zeta), \qquad \varphi = \cos\alpha\, s - \sin\alpha\, \zeta. \tag{1}$$

Dimension of spiral shell:

$$-l/2 \le R\, s \le l/2, \qquad -a/2 \le R\, \zeta \le a/2. \tag{2}$$

Here R — radius of cylinder, α — angle of inclination of spiral.

Equations of classical shell theory.

$$\nabla \cdot \underline{\underline{T}} = \rho \ddot{\underline{u}}, \qquad \nabla \cdot \underline{\underline{M}} + \underline{\underline{T}}_{\times} = 0. \tag{3}$$

$$\underline{\varphi} = -\underline{n} \times (\nabla \underline{u}) \cdot \underline{n}, \qquad \underline{\underline{T}} = \underline{\underline{T}} \cdot \underline{\underline{a}} + \underline{N}\,\underline{n}. \tag{4}$$

$$\underline{\underline{T}} \cdot \underline{\underline{a}} + \frac{1}{2}(\underline{\underline{M}} \cdot\cdot \underline{\underline{b}})\underline{\underline{c}} = {}^4\underline{\underline{A}} \cdot\cdot \underline{\underline{\varepsilon}}, \qquad \underline{\underline{M}}^T = {}^4\underline{\underline{C}} \cdot\cdot \underline{\underline{\kappa}}. \tag{5}$$

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\left((\nabla \underline{u}) \cdot \underline{\underline{a}} + \underline{\underline{a}} \cdot (\nabla \underline{u})^T\right), \qquad \underline{\underline{\kappa}} = (\nabla \underline{\varphi}) \cdot \underline{\underline{a}} + \frac{1}{2}\left((\nabla \underline{u}) \cdot\cdot \underline{\underline{c}}\right) \underline{\underline{b}}. \tag{6}$$

Here $\underline{\underline{T}}$, $\underline{\underline{M}}$ — force tensor and moment tensor, ρ — surface mass density, $\underline{u}$, $\underline{\varphi}$ — displacement vector and turn vector, $\underline{\underline{\varepsilon}}$, $\underline{\underline{\kappa}}$ — strain tensors, ${}^4\underline{\underline{A}}$, ${}^4\underline{\underline{C}}$ — stiffness tensors, $\underline{\underline{a}}$ — unit tensor in tangential plane, $\underline{n}$ — normal unit vector, $\underline{\underline{b}} = -\nabla \underline{n}$, $\underline{\underline{c}} = -\underline{\underline{a}} \times \underline{n}$.

Approximate equations of motion of thin shell.

Let us suppose, that

$$\underline{\underline{\varepsilon}} = 0. \tag{7}$$

Equation of motion:

$$\left(\sin^2\alpha\frac{\partial^4}{\partial s^4} + \cos^2\alpha\frac{\partial^4}{\partial \zeta^4} - \frac{1}{4}\frac{\partial^4}{\partial s^2 \partial \zeta^2}\right)\left[\frac{D}{\rho R^4}(\tilde{\triangle} + 1)^2 w + \ddot{w}\right] - \\ -\frac{\sin^2 2\alpha}{4}\tilde{\triangle}\ddot{w} = 0. \tag{8}$$

Equation of indissolubleness:

$$\sin 2\alpha\,\frac{\partial^2 w}{\partial s \partial \zeta} + \sin^2\alpha\,\frac{\partial^2 w}{\partial s^2} + \cos^2\alpha\,\frac{\partial^2 w}{\partial \zeta^2} = 0. \tag{9}$$

Here w — displacement along vector $\underline{n}$, D — bending stiffness.

Solution of motion equation, satisfying indissolubleness equation:

$$w(s, \zeta, t) = W(s, \zeta)\, e^{i\omega t}. \tag{10}$$

$$W = \sum_{j=1}^{3} [(A_j^s(p_j s + q_j \zeta) + B_j^s) \sin[\lambda_j(\cos\alpha\, s - \sin\alpha\, \zeta)] +$$

$$+ (A_j^c(p_j s + q_j \zeta) + B_j^c) \cos[\lambda_j(\cos\alpha\, s - \sin\alpha\, \zeta)]],$$

$$p_j = \sin\alpha - \beta_j, \qquad q_j = \cos\alpha + \beta_j, \tag{11}$$

$$\beta_j = \frac{2\cos 2\alpha\, \Omega^2}{9\cos\alpha\,(\lambda_j^4 + (\Omega^2 - 1)\lambda_j^2 + 2\Omega^2)}.$$

A_j^s, B_j^s, A_j^c, B_j^c — constants, λ_j — roots of characteristic equation

$$\lambda^6 - 2\lambda^4 + (1 - \Omega^2)\lambda^2 - \frac{4}{3}\Omega^2 = 0, \qquad \Omega = \sqrt{\frac{\rho R^4}{D}}\,\omega. \tag{12}$$

Boundary conditions:

$$\underline{u}\left(\frac{l}{2R},\frac{a}{2R},t\right)=0,\qquad \underline{u}\left(-\frac{l}{2R},\frac{a}{2R},t\right)=0,$$
$$\underline{u}\left(\frac{l}{2R},-\frac{a}{2R},t\right)=0,\qquad \underline{u}\left(-\frac{l}{2R},-\frac{a}{2R},t\right)=0. \tag{13}$$

Discussion of results.

Eigenfrequencies are depend on three parameters:

$$\Omega_n=\Omega_n\left(\alpha,\frac{l}{R},\frac{a}{R}\right),\qquad n=1,\,2,\,3,\,\ldots \tag{14}$$

Let us consider two shells, having different physical and geometrical characteristics, but identical parameters α, l/R, a/R. Then

$$\forall\, n:\qquad \Omega_n^{(1)}=\Omega_n^{(2)}\qquad\Rightarrow\qquad \frac{\omega_n^{(1)}}{\omega_n^{(2)}}=\sqrt{\frac{D_1\rho_2R_2^4}{D_2\rho_1R_1^4}}. \tag{15}$$

Acoustical and optical methods of measuring of eigenfrequencies of micro-objects

I. Sokolov (*A.F. Ioffe Physico-Technical Institute RAS, St.Petersburg*)

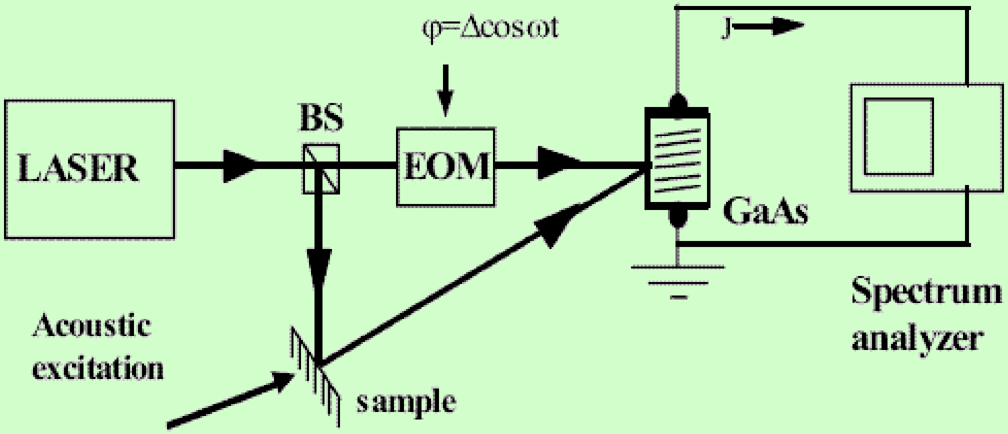


Figure 8: Experimental setup of the interferometer with adaptive photodetector.

- Limited frequency range.
- Laser ray is a spot of certain diameter (not a point).
- Laser ray is focused with some error.
- Frequencies of what object is measured?

Conclusion: We can not measure eigenfrequencies of nano-objects. We can measure eigenfrequencies of nano-objects on the micro-substrate.

Mechanical problems:

- At what correlation between nano-object and substrate sizes we can obtain information about nano-object?
- What kind of connection of nano-object on substrate and connection of the substrate on the device frame is optimal?
- Determination of mechanical properties of nano-object by the frequencies of system including nano-object and substrate.

Atomic Force Microscope

A. Ankudinov, A. Titkov (*A.F. Ioffe Physico-Technical Institute*)

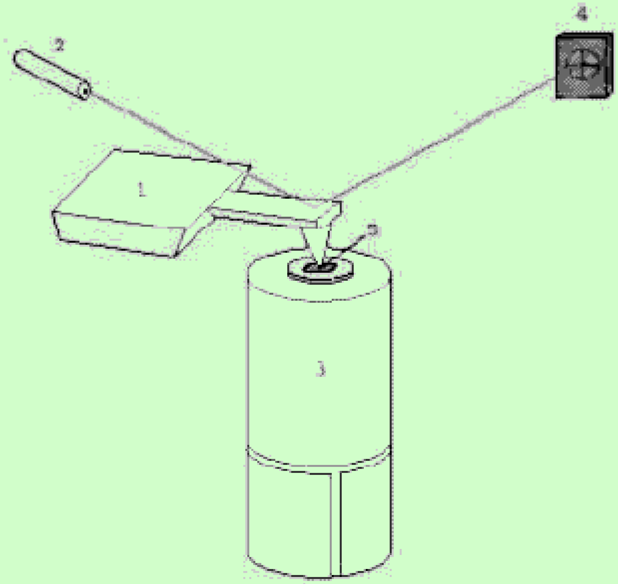


Figure 10: Atomic Force Microscope.

- Limited frequency range.
- Needle of cantilever has certain curvature radius (not zero).
- Influence of substrate on eigenfrequencies of nano-object.
- Needle of cantilever acts on nano-object.

Conclusion: In principle, using AFM we can measure frequencies of nano-objects. In fact, using AFM we measure frequencies of the system of nano-objects and cantilever.

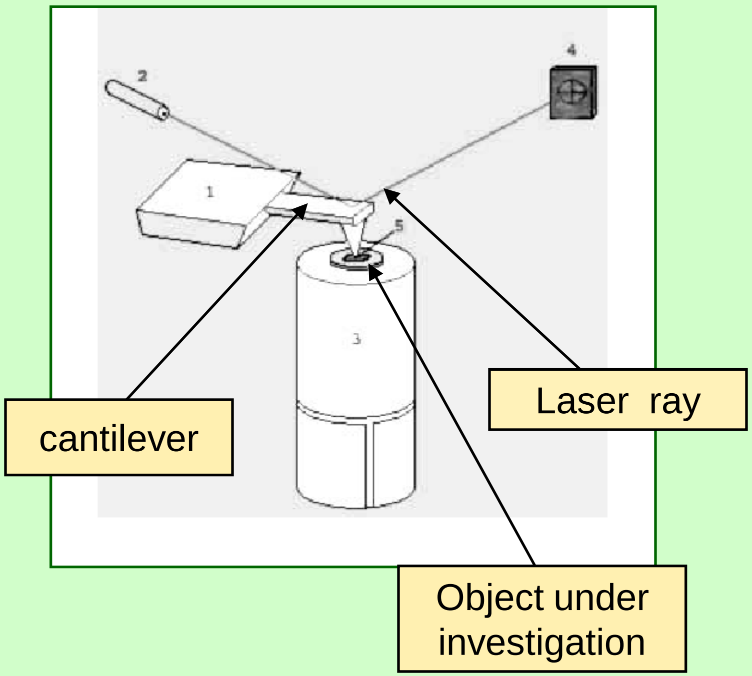
Mechanical problems:

- Determination of mechanical properties of nano-object by the frequencies of system including nano-object and cantilever.
- Can we extract eigenfrequencies of nano-object from the spectrum of system including nano-object and cantilever?

Method of experimental determination of mechanical characteristics of nano-objects

D.A. Indeitsev, E.A. Ivanova, N.F. Morozov

Measuring device:
atomic force microscope



Method:

Mechanical characteristics of nano-object are determined by the eigenfrequencies.

Main difficulty:

Eigenfrequencies of a system of nano-object and cantilever, (not eigenfrequencies of the nano-object) are measured.

Mechanics problem:

Development of a method of determination of characteristics of nano-objects in conditions when frequencies of system of nano-object and cantilever and mechanical parameters of cantilever are known.

New results:

Method of an experimental research of nano-objects with own dynamics is proposed.

Cantilever and nano-rod

D. Indeitsev, E. Ivanova, N. Morozov

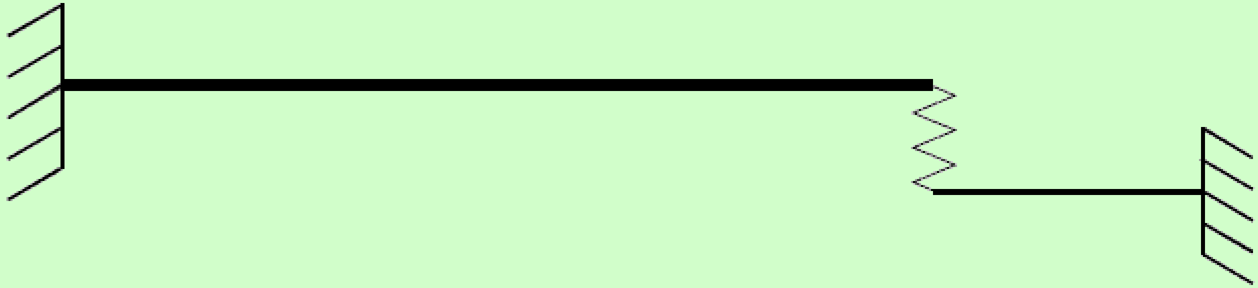


Figure 11: Cantilever (left) and nano-rod (right).

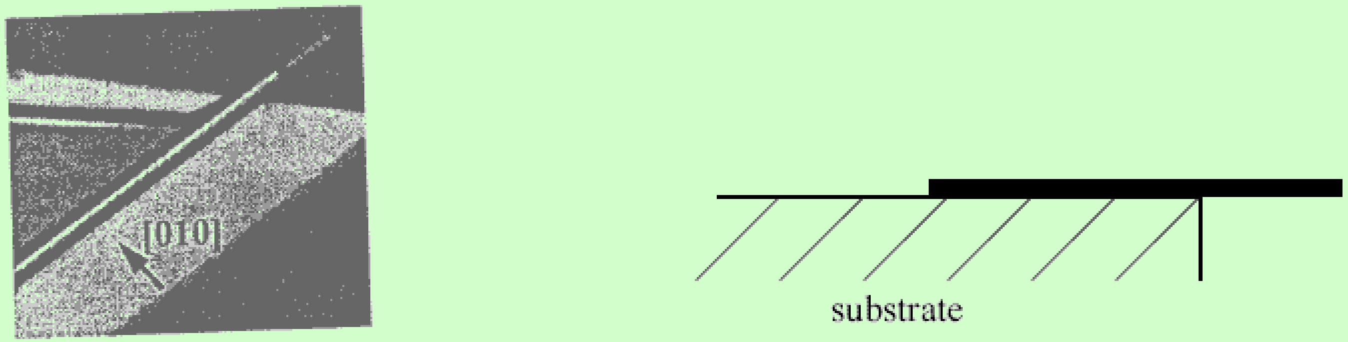


Figure 12: Experiments by V. Prinz (Novosibirsk).

Free vibrations of system

Cantilever:

$$D_1 u^{IV} + \rho_1 \ddot{u} = 0, \qquad u(0) = 0, \quad u'(0) = 0, \quad u''(L_1) = 0.$$

Nano-rod:

$$D_1 v^{IV} + \rho_1 \ddot{v} = 0, \qquad v(0) = 0, \quad v'(0) = 0, \quad v''(L_2) = 0.$$

Interaction of cantilever and nano-rod:

$$D_1 u'''(L_1) = C(u(L_1) - v(L_2)), \qquad D_2 v'''(L_2) = -C(u(L_1) - v(L_2)).$$

Let us note $\lambda^2 = \sqrt{\frac{\rho_1}{D_1}}\omega$, $\mu^2 = \sqrt{\frac{\rho_2}{D_2}}\omega$. Frequency equation:

$$\Big[1 + \cos(\lambda L_1)\mathrm{ch}(\lambda L_1)\Big]\Big(1 + \cos(\mu L_2)\mathrm{ch}(\mu L_2) +$$
$$+ \frac{C}{D_2 \mu^3}[\sin(\mu L_2)\mathrm{ch}(\mu L_2) - \cos(\mu L_2)\mathrm{sh}(\mu L_2)]\Big) +$$
$$+ \frac{C}{D_1 \lambda^3}[\sin(\lambda L_1)\mathrm{ch}(\lambda L_1) - \cos(\lambda L_1)\mathrm{sh}(\lambda L_1)]\,(1 + \cos(\mu L_2)\mathrm{ch}(\mu L_2)) = 0.$$

Force vibrations of system

$$u(0) = A\sin(\Omega t), \qquad A = \mathrm{const}.$$

Solution has following form:

$$u = \left[P_1\cos(\lambda_* x_1) + P_2\sin(\lambda_* x_1) + P_3\mathrm{ch}(\lambda_* x_1) + P_4\mathrm{sh}(\lambda_* x_1)\right]\sin(\Omega t),$$

$$v = \left[Q_1\cos(\mu_* x_2) + Q_2\sin(\mu_* x_2) + Q_3\mathrm{ch}(\mu_* x_2) + Q_4\mathrm{sh}(\mu_* x_2)\right]\sin(\Omega t),$$

where $\lambda_*^2 = \sqrt{\dfrac{\rho_1}{D_1}}\Omega, \quad \mu_*^2 = \sqrt{\dfrac{\rho_2}{D_2}}\Omega.$

Dynamical damping of cantilever vibrations:

$$u(L_1, t) = 0.$$

Equation for determination Ω:

$$1+\cos(\mu_* L_2)\text{ch}(\mu_* L_2)+\frac{C}{D_2\mu_*^3}[\sin(\mu_* L_2)\text{ch}(\mu_* L_2)-\cos(\mu_* L_2)\text{sh}(\mu_* L_2)] = 0.$$

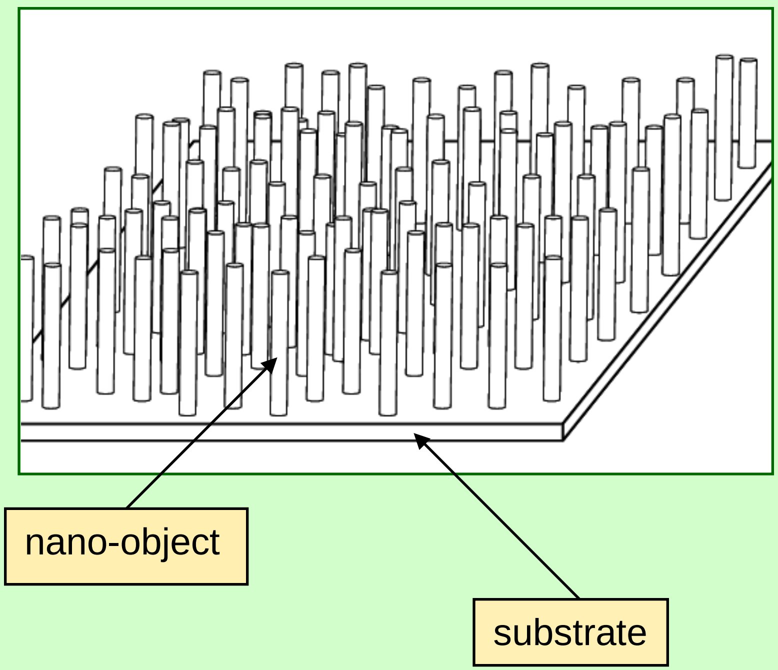
Following estimation takes place: $\dfrac{C}{D_1\lambda^3} \ll \dfrac{C}{D_2\mu^3}$.

Hence, Ω_n is very near eigenfrequencies of nano-rod.

Method of determination of nano-objects characteristics by the eigenfrequencies of a system consisting of nano-objects and substrate

V.A. Eremeyev, E.A. Ivanova, N.F. Morozov

Investigated object:
regular structure of identical
nano-objects on a micro-substrate

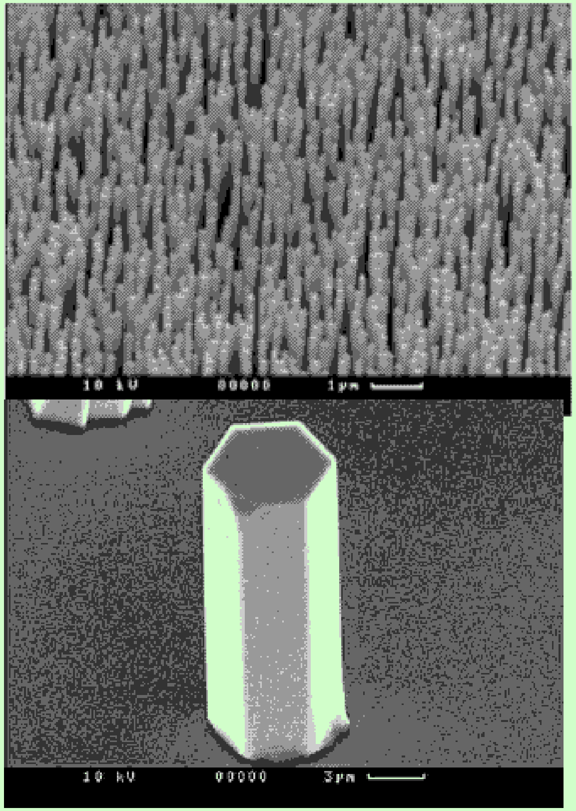


Results of measurements:
Eigenfrequencies of system of nano-objects and micro-substrate are measured.

Mechanics problem :
To determine mechanical characteristics of nano-objects in conditions when mechanical parameters of a micro-substrate and eigenfrequencies of system are known.

Determination of mechanical properties of nanoobjects

We propose the new method of determination of mechanical properties of one nanocrystal or nanotube by experiments on oriented nanocrystals or nanotubes arrays grown on substrate



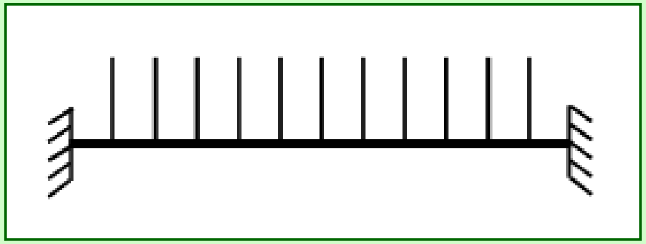
Nanocrystals on (11$\bar{2}$0) sapphire grown by multi-step pulsed laser deposition and one nanocrystal. Diameter is 150-250 nm, length is 1000 nm

Method:

1. Frequencies of a substrate with nano-objects are measured.
2. Frequencies of a substrate without nano-objects are measured.
3. By comparison of two these spectra, eigenfrequencies of nano-objects are separated from a frequency spectrum of a substrate with nano-objects.

Analytical result:

It has been proved, that eigenfrequencies of nano-objects can be separated from a frequency spectrum of a substrate with nano-objects.



Equation of substrate vibrations:

$$u^{IV} - \frac{ND\mu}{Cg(\mu H)L}u'' + \frac{\rho_1}{C}\left(1 + N\frac{\rho_2 H}{\rho_1 L}\right)\ddot{u} = 0.$$

$$g(\mu H) = \frac{1 + \cos(\mu H)\mathrm{ch}(\mu H)}{\sin(\mu H)\mathrm{ch}(\mu H) - \cos(\mu H)\mathrm{sh}(\mu H)}.$$

$$g(\mu H) \neq 0$$

Eigenfrequencies of substrate.

Nano-objects do not move!

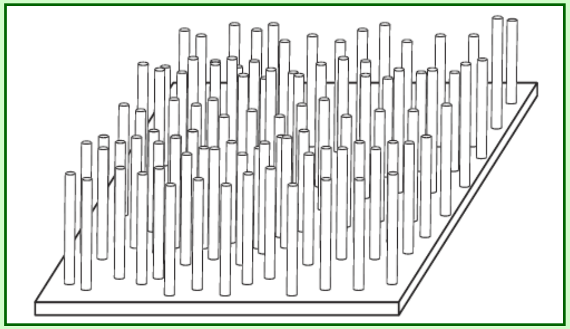
Eigenfrequencies of nano-object:

$$1 + \cos(\mu H)\mathrm{ch}(\mu H) = 0, \qquad \mu = \sqrt[4]{\frac{\rho_2}{D}}\sqrt{\omega}.$$

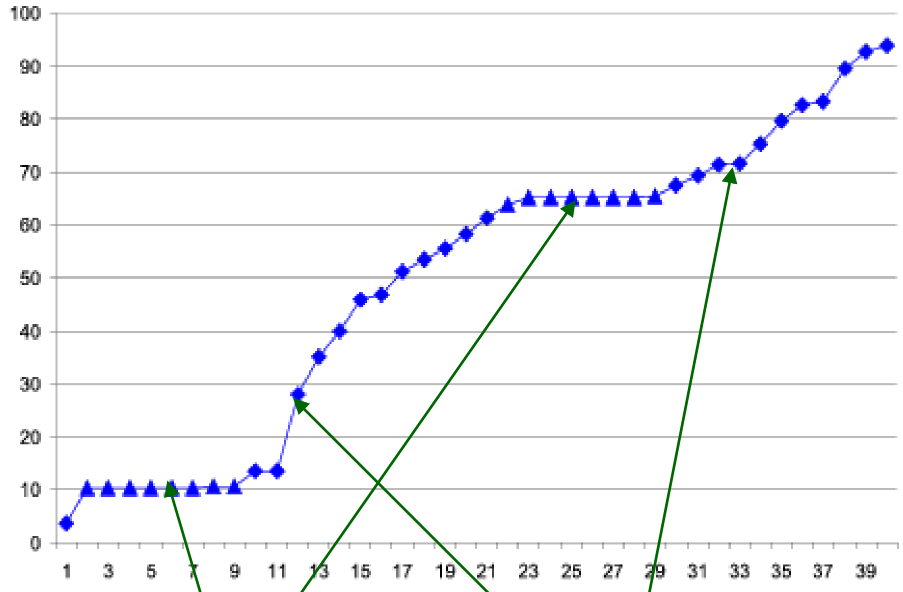
$$g(\mu H) = 0 \Rightarrow \mathrm{u} = 0$$

substrate does not move!

Calculation of eigenfrequencies and eigenforms by numerical method. 3D statement of problem



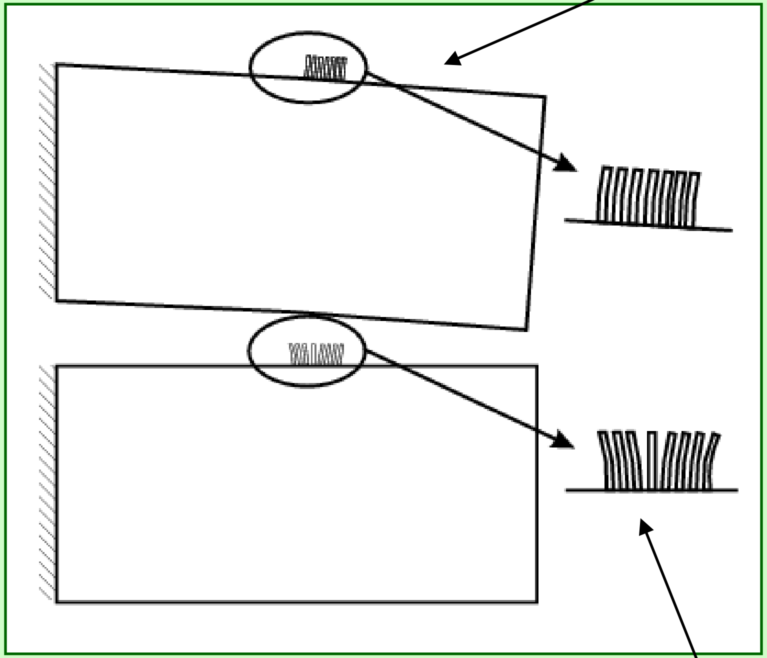
Numerical results:



nano-object frequencies

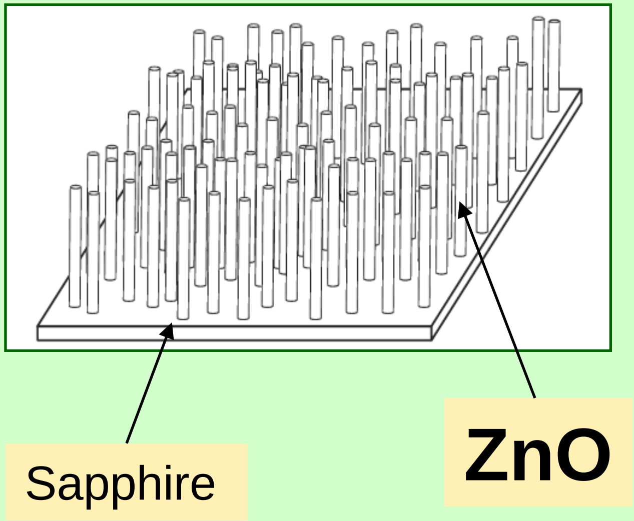
substrate frequencies

substrate eigenforms



nano-object eigenforms

Calculation of eigenfrequencies.
3D statement of problem



substrate with nano-objects (GHz)	Substrate (GHz)
0.036494	0.036594
0.103909	
0.103971	
0.104039	
0.104106	
0.104226	
0.104322	
0.104467	
0.104612	
0.134652	0.134973
0.136246	0.136017
0.280004	0.280137
0.350831	0.352308

Nano-object (GHz)

0.10797

Nano-crystals ZnO

High: 1.5-3.0 mkm

Diameter: 30-100 nm