Short biography and scientific results of P. A. Zhilin*

Pavel Andreevich Zhilin was the Head of the Department of Theoretical Mechanics at Saint Petersburg Polytechnical University, Head of the laboratory "Dynamics of Mechanical Systems" at the Institute for Problems in Mechanical Engineering of Russian Academy of Sciences, member of the Russian National Committee for Theoretical and Applied Mechanics, member of the International Society of Applied Mathematics and Mechanics (GAMM), member of Guidance Board Presidium for Applied Mechanics Ministry of Higher Education RF, full member of Russian Academy of Sciences for Strength Problems. He was an author of more than 200 scientific papers, monographs "Second-rank Vectors and Tensors in 3-dimentional space" (2001), "Theoretical mechanics: fundamental laws of mechanics" (2003). Sixteen PhD theses and six Professorial theses were defended under his supervision.

P.A. Zhilin was born on February 8th, 1942, in the town of Velikiy Ustyug in Vologda region, where his family found themselves during the war. Pavel Zhilin spent his childhood in the towns of Volkhov and Podporozhie, where his father, Andrey Pavlovich Zhilin, worked. Andrey P. Zhilin was a power engineering specialist, and at that time the chief engineer at the coordinated hydroelectric system of Svir river. Zoya Alexeevna Zhilina, mother of Pavel A. Zhilin, was bringing up the sons and kept the house. In 1956 Andrey P. Zhilin was assigned to the position of the chief power engineering specialist at the Soviet Union Trust "HydroElectroMontage", and the family moved to Leningrad. The elder brother, Sergey Andreevich Zhilin, followed in his father's footsteps, became an engineer and now participates in creating high-voltage electric apparatus. In 1959 P.A. Zhilin left the secondary school and entered Leningrad Polytechnical Institute. Yet at school Pavel Zhilin met his future wife, Nina Alexandrovna, who was his faithful friend and helpmate all his life long. While studying at the institute P.A. Zhilin became keen on table tennis and was a captain of the student and later institute team for many years. Not once did the team win different student and sport collectives championships. P.A. Zhilin got a qualification of the candidate master of sports (the highest qualification in this sport discipline at that time).

In the period of 1959–1965 P.A. Zhilin studied at Leningrad Polytechnical Institute

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in the Department of "Mechanics and Control Processes" at the Faculty of Physics and Mechanics. Later on his daughter, Olga Zhilina, graduated from the same Department. After graduation, P.A. Zhilin got a qualification of engineer-physicist in "Dynamics and Strength of Machines" speciality, and from 1965 to 1967 worked as an engineer at water turbine strength department in the Central Boiler Turbine Institute. In 1967 he accepted a position of Assistant Professor at the Department of "Mechanics and Control Processes", later he worked there as a senior researcher, an Associate Professor and a Full Professor. The founder of the Chair was Anatoliy Isaakovich Lurie, Doctor of Technical Sciences, Professor, corresponding member of USSR Academy of Sciences, world-famous scientist. P.A. Zhilin became the closest disciple of A.I. Lurie and spent many hours working together with him. Scientific ideology of P.A. Zhilin was developing to a great extent under the influence of A.I. Lurie. P.A. Zhilin got his PhD degree in Physical and Mathematical Sciences in 1968 (the topic of his thesis was "The theory of ribbed shells"), Professor of Physical and Mathematical Sciences since 1984 (the topic of his Professorial thesis was "The theory of simple shells and its applications"), Professor at the Department of "Mechanics and Control Processes" since 1989. In 1974–1975 P.A. Zhilin worked as a visiting researcher at the Technical University of Denmark. While working in the Department of "Mechanics and Control Processes", P.A. Zhilin delivered lectures on analytical mechanics, theory of oscillations, theory of shells, tensor analysis, continuum mechanics. In 1988 he was invited in the Yarmuk University (Jordan) to set a course of continuum mechanics at the Faculty of Physics. At the same time P.A. Zhilin actively carried out scientific work in the field of theory of plates and shells, nonlinear rod theory, theory of elasticity, continuum mechanics. He gained three certificates of invention in the area of vibroinsulation and hydroacoustics, he was awarded with the Inventor of the USSR insignia.

Since 1989 P.A. Zhilin was the Head of Department of Theoretical Mechanics. In the period of his direction five of his colleagues defended their Professorial theses, for the four of them P.A. Zhilin was a scientific advisor. While working in the Department of Theoretical Mechanics P.A. Zhilin stationed and read original courses on tensor algebra, rational mechanics, and the rod theory. During this period of time Pavel Zhilin worked hard in the field of investigating and developing foundations of mechanics. His investigations on spinor motions in mechanics and physics, phase transitions and phenomena of inelasticity, electrodynamics from the positions of rational mechanics, logical foundations of mechanics relate to this period. Since 1994 Pavel Zhilin was the Head of "Dynamics of Mechanical Systems" laboratory at the Institute for Problems in Mechanical Engineering of Russian Academy of Sciences. Since 1999 he was a member of the scientific committee of the Annual International Summer School – Conference "Advanced Problems in Mechanics", held by the Institute for Problems in Mechanical Engineering.

Pavel Andreevich Zhilin died on 4th of December, 2005. His track has become a part of history of science. It is difficult to overestimate his influence on his disciples, colleagues, and all who were lucky to know him personally. He had an extraordinary ability to inspire interest to science, to give you a fresh unexpected look at the world around. P.A. Zhilin was a man of heart, a responsive, kind person, who found time for everyone, always giving his full support and benefit of his wise advice. One was amazed by his remarkable human qualities, his absolute scientific and human honesty. Being his disciples we are grateful to life for the chance to have known such a wonderful person and an outstanding scientist, who became for us an embodiment of spirituality.

Scientific results

Theory of shells

Zhilin's early works, Ph.D. and Professor theses were devoted to the development of the theory of shells. When Zhilin started his research in this area, there existed no general theory of shells. For each class of shell-type constructions there were developed particular independent theories: the theory of thin single-layer shells; the theory of engineering anisotropic shells; the theory of ribbed shells; the theory of thin multi-layer shells; the theory of perforated shells; the theory of cellular shells; the theory of thick single-layer shells, and many others. Within each theory one could distinguish several versions, which differed in basic assumptions as well as in final equations. The theories of shells are still being developed since the science gives birth to new constructions that can not be described within the existing variety of theories. Zhilin created (1975–1984) the general nonlinear theory of thermoelastic shells, whose way of construction fundamentally differs from the one of all known versions of shell theories, and can be easily generalised for any shell-like constructions and other objects of continuum mechanics. This approach is comprehensively described in work [1].

1. Zhilin P.A. Applied mechanics. Foundations of theory of shells. Tutorial book. St. Petersburg State Polytechnical University. 2006. 167 p. (In Russian).

Discretely stiffened thermoelastic shells

The general theory of discretely stiffened thermoelastic shells was developed (1965–1970) [1, 3] and applied to the following practical problems: the calculation of the high-pressure water turbine scroll of Nurek hydropower station [2] and of the vacuum chamber of the thermonuclear Tokamak 20 Panel [4].

There was proposed (1966) a variant of the Steklov-Fubini method for differential equations, whose coefficients have singularities of δ-function type. The method allowed to find the solution in an explicit form for the problem of axisymmetric deformation of a discretely stiffened cylindrical shell [5].

1. Zhilin P.A. General theory of ribbed shells // Trudi CKTI (Transactions of Central Boiler Turbine Institute). 1968. No. 88. Pp. 46–70. (In Russian.)

2. Zhilin P.A., Mikheev V.I. Toroidal shell with meridional ribs for design of hydroturbine spirals. // Trudi CKTI (Transactions of Central Boiler Turbine Institute). 1968. No. 88. Pp. 91–99. (In Russian.)

3. Zhilin P.A. Linear theory of ribbed shells // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1970. No. 4. Pp. 150–162. (In Russian.)

4. Zhilin P.A., Konyushevskaya R.M., Palmov V.A., Chvartatsky R.V. On design of the stress-strain state of discharge chambers of Tokamak Panels. Leningrad, NIIEFA (Research Institute of Electro-physical apparatuses), P-OM-0550. 1982. Pp. 1–13. (In Russian.)

5. Zhilin P.A. Axisymmetric deformation of a cylindrical shell, supported by frames // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1966, No. 5, pp. 139–142. (In Russian.)

A new formulation for the second law of thermodynamics for the case of thin surfaces

A new formulation for the second law of thermodynamics was proposed (1973) [1–4] by means of the combination of two Clausius–Duhem–Truesdell type inequalities. This formulation deals with a thin surface, each side of which has its own temperature and entropy. So, the formulation contains two entropies, two internal temperature fields, and two external temperature fields. Apart from the theory of shells this elaboration of the second law of thermodynamics is also useful for the solid-state physics when studying the influence of skin effects on properties of solids, as well as for the description of interfaces between different phases of a solid.

1. Zhilin P.A. Mechanics of Deformable Surfaces. The Danish Center for Appl. Math and Mech. Report N 89. 1975. P. 1–29.

2. Zhilin P.A. Mechanics of Deformable Cosserat Surfaces and Shell Theory. The Danish Center for Appl. Math and Mech. Annual report. 1975.

3. Zhilin P.A. Mechanics of deformable enriched surfaces // Transactions of the 9th Soviet conference on the theory of shells and plates. Leningrad, Sudostroenie. 1975. Pp. 48–54. (In Russian.)

4. Zhilin P.A. Mechanics of Deformable Directed Surfaces // Int. J. Solids Structures. 1976. Vol. 12. P. 635–648.

Generalization of the classical theory of symmetry of tensors

An important addition is made (1977) to the tensor algebra, namely the concept of oriented tensors, i.e. tensor objects which depend on orientation in both a three-dimensional space, and in its subspaces. The theory of symmetry [1, 2] is formulated for oriented tensors, and it generalises the classical theory of symmetry, which applies to the Euclidean tensors only. It was shown that the application of the classical theory, for example, to axial tensors, i.e. objects dependent on orientation in a 3D space, leads to wrong conclusions. The proposed theory is needed to obtain the constitutive equations for shells and other multipolar media, as well as when studying ionic crystals.

1. Zhilin P.A. General theory of constitutive equations in the linear theory of elastic shells // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1978. No. 3. Pp. 190. (In Russian.)

2. Zhilin P.A. Basic equations of non-classical theory of shells // Dinamika i prochnost mashin (Dynamics and strength of machines.) Trudi LPI (Proceedings of Leningrad Polytechical Institute.) N 386. 1982. . 29–46. (In Russian.)

The general nonlinear theory of thermoelastic shells

The general nonlinear theory of thermoelastic shells is created (1975–1984). The way of its construction fundamentally differs from all known versions of shell theories and can be easily extended to any shell-like constructions and other objects of continuum mechanics. Its key feature is that it allows studying shell-like objects of a complex internal structure, i.e. when traditional methods of construction of the theory of shells are not applicable [1–11]. For shells of constant thickness, made of isotropic material, the new method gives results that are in accordance with those of the classical methods and perfectly coincide with the results of three-dimensional elasticity theory for the case of any external forces, including point loads.

1. Zhilin P.A. Two-dimensional continuum. Mathematical theory and physical interpretations // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1972. N 6. Pp. 207–208. (In Russian.)

2. Zhilin P.A. Modern handling of the theory of shells // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1974. N 4. (In Russian.)

3. Zhilin P.A. Mechanics of Deformable Surfaces. The Danish Center for Appl. Math and Mech. Report N 89. 1975. P. 1–29.

4. Zhilin P.A. Mechanics of Deformable Cosserat Surfaces and Shell Theory. The Danish Center for Appl. Math and Mech. Annual report. 1975.

5. Zhilin P.A. Mechanics of deformable enriched surfaces // Transactions of the 9th Soviet conference on the theory of shells and plates. Leningrad, Sudostroenie. 1975. Pp. 48–54. (In Russian.)

6. Zhilin P.A. Mechanics of Deformable Directed Surfaces // Int. J. Solids Structures. 1976. Vol. 12. P. 635–648.

7. Zhilin P.A. General theory of constitutive equations in the linear theory of elastic shells // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1978. No. 3. Pp. 190. (In Russian.)

8. Zhilin P.A. A new method for the construction of theory of thin elastic shells // . . . Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1978. No. 3. (In Russian.)

9. Zhilin P.A. Direct construction of the theory of shells basing on physical principles // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1980. No. 3. Pp. 179. (In Russian.)

10. Zhilin P.A. Basic equations of non-classical theory of shells // Dinamika i prochnost mashin (Dynamics and strength of machines.) Trudi LPI (Proceedings of Leningrad Polytechical Institute.) N 386. 1982. . 29–46. (In Russian.)

11. Altenbach H., Zhilin P.A. General theory of elastic simple shells // Advances in mechanics — Warszawa, Polska. 1988. N 4. P. 107–148. (In Russian).

Elimination of a paradox in the problem of bending deflection of a round plate

The exact analytical solution is given (1982) for the problem of final displacements of a round plate [1, 2]. The solution explains a well-known paradox which was described in handbooks and assumed that the deflection of a membrane, i.e. a plate with zero beam stiffness, was less than the deflection calculated with non-zero beam stiffness taken into account. (The problem considers a round plate with its edges fixed and loaded by transversal pressure, whose magnitude makes the application of the linear theory incorrect. The latter one overestimates the deflection approximately 25 times). Later the idea of works [1, 2] was used for calculation of an electrodynamic gate [3].

1. Zhilin P.A. Axisymmetric bending of a flexible circular plate under large displacements // Vichislitelnie metodi v mekhanike i upravlenii (Numerical methods in mechanics and control theory). Trudi LPI (Proceedings of Leningrad Polytechical Institute.) N 388. 1982. . 97–106. (In Russian.)

2. Zhilin P.A. Axisymmetrical bending of a circular plate under large displacements // Izvestiya AN SSSR, Mekhanika tverdogo tela (Transactions of the Academy of Sciences of the USSR, Mechanics of Solids). 1984. No. 3. Pp. 138–144. (In Russian.)

3. Venatovsky I.V., Zhilin P.A., Komyagin D.Yu. Inventor's certificate No. 1490663 with priority from 02.11.1987. (In Russian.)

Critical surveys

1. Zhilin P.A. The view on Poisson's and Kirchhoff's theories of plates in terms of modern theory of plates // Izvestia RAN. Mechanika Tverdogo Tela (Mechanics of Solids). 1992. N 3. P. 48–64. (In Russian).

2. Zhilin P.A. On the classical theory of plates and the Kelvin-Teit transformation // Izvestia RAN. Mechanika Tverdogo Tela (Mechanics of Solids). 1995. N 4. P. 133–140. (In Russian).

3. Altenbach H., Zhilin P.A. The Theory of Simple Elastic Shells // in Critical Review of The Theories of Plates and Shells and New Applications, ed. by H. Altenbach and R. Kienzler. Berlin, Springer. 2004. P. 1–12.

The theory of rods

The dynamic theory of thin spatially curvilinear rods and naturally twisted rods is developed (1987–2005). The proposed theory includes all known variants of theories of rods, but it has wider domain of application. A significant part of the work is devoted to the analysis of a series of classical problems, including those whose solutions demonstrate paradoxes. The results of the theory of rods and its applications are presented in the most complete way in work [1].

1. Zhilin P.A. Applied mechanics. Theory of thin elastic rods. Tutorial book. St. Petersburg State Polytechnical University. 2006. 98 p. (In Russian). To be published.

General nonlinear theory of rods and its applications to the solution of particular problems

Basing on method developed in the theory of shells, the general nonlinear theory of flexible rods is formulated (1987), where all the basic types of deformation: bending, torsion, tension, transversal shear are taken into account. Use of the rotation (turn) tensor allowed to write down the equations in a compact form, convenient for the mathematical analysis. In contradistinction to previous theories, the proposed theory describes the experimentally discovered Pointing effect (the contraction of a rod under torsion). The developed theory was applied to analyse the series of particular problems [2, 3]. A new method [4–6] was suggested (2005) for the construction of elastic tensors, and their structure has been determined. In this work the new theory of symmetry of tensors, determined in the space with two independent orientations, is essentially used. All elastic constants were found for plane curvilinear rods.

1. Goloskokov D.P., Zhilin .. General nonlinear theory of elastic rods with application to the description of the Pointing effect // Deposited in VINITI No. 1912-V87. Dep. 20 p. (In Russian.)

2. Zhilin P.A., Tovstik T.P. Rotation of a rigid body based on an inertial rod // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1995. N 458. P. 78–83. (In Russian).

3. Zhilin P.A., Sergeyev A.D., Tovstik T.P. Nonlinear theory of rods and its application // Proc. of XXIV Summer School - Seminar "Nonlinear Oscillations in Mechanical Systems". St. Petersburg. 1997. P. 313–337. (In Russian).

4. Zhilin P.A. Theory of thin elastic rods //Lecture at XXXIII Summer School - Conference "Advanced Problems in Mechanics". St. Petersburg, Russia. 2005. Current book. Vol. 1. (In Russian).

5. Zhilin P.A. Nonlinear Theory of Thin Rods // Lecture at XXXIII Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg, Russia. 2005. Current book. Vol. 2.

6. Zhilin P.A. Applied mechanics. Theory of thin elastic rods. Tutorial book. St. Petersburg State Polytechnical University. 2006. 98 p. (In Russian). To be published.

The Euler elastica

The famous Euler elastica [1–5] was considered (1997–2005) and it was shown that apart from the known static equilibrium configurations there exist also dynamic equilibrium configurations. In the latter case, the form of elastic curve remains the same, and the bent rod rotates about the vertical axe. The energy of deformation does not change in this motion. Note that we do not speak about the rigid motion of a rod, since the clamped end of the rod remains fixed. This means that the curvilinear equilibrium configuration in the Euler elastica is unstable, contrary to the common point of view. On the other hand, this conclusion is not confirmed by experiments. Thus there appears a paradox requiring its explanation.

1. Zhilin P.A., Sergeyev A.D., Tovstik T.P. Nonlinear theory of rods and its application // Proc. of XXIV Summer School - Seminar “Nonlinear Oscillations in Mechanical Systems”. St. Petersburg. 1997. P. 313–337. (In Russian).

2. Zhilin P.A. Dynamic Forms of Equilibrium Bar Compressed by a Dead Force // Proc. of 1st Int. Conf. Control of Oscillations and Chaos. Vol. 3. 1997. P. 399–402.

3. Zhilin P.A. Theory of thin elastic rods //Lecture at XXXIII Summer School - Conference “Advanced Problems in Mechanics”. St. Petersburg, Russia. 2005. Current book. Vol. 1. (In Russian).

4. Zhilin P.A. Nonlinear Theory of Thin Rods // Lecture at XXXIII Summer School – Conference “Advanced Problems in Mechanics”. St. Petersburg, Russia. 2005. Current book. Vol. 2.

5. Zhilin P.A. Applied mechanics. Theory of thin elastic rods. Tutorial book. St. Petersburg State Polytechnical University. 2006. 98 p. (In Russian). To be published.

Nikolai paradox

The Nikolai paradox [1–7] is analysed (1993-2005). The paradox appears when a rod is subjected to the torsion by means of the torque applied to its end. The experiment shows that the torsion torque stabilises the rod, which is in the major contradiction with the theory. It is shown [6], that one may avoid the mentioned paradox if to choose a special constitutive equation for the torque. The torque has to depend in a special way on the rotational velocity. This dependence is not related to the existence (or absence) of the internal friction in the rod.

1. Zhilin P.A., Sergeyev A.D. Twisting of an elastic cantilever rod by a torque subjected at a free end. St. Petersburg State Technical University. 1993. 32 p. (In Russian).

2. Zhilin P.A., Sergeyev A.D. Experimental investigation of the stability of a cantilever rod under torsion deforming // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1993. N 446. P. 174–175. (In Russian).

3. Zhilin P.A., Sergeyev A.D. Equilibrium and stability of a thin rod subjected to a conservative moment // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1994. N 448. P. 47–56. (In Russian).

4. Zhilin P.A., Sergeyev A.D., Tovstik T.P. Nonlinear theory of rods and its application // Proc. of XXIV Summer School - Seminar “Nonlinear Oscillations in Mechanical Systems”. St. Petersburg. 1997. P. 313–337. (In Russian).

5. Zhilin P.A. Theory of thin elastic rods //Lecture at XXXIII Summer School - Conference “Advanced Problems in Mechanics”. St. Petersburg, Russia. 2005. Current book. Vol. 1. (In Russian).

6. Zhilin P.A. Nonlinear Theory of Thin Rods // Lecture at XXXIII Summer School – Conference “Advanced Problems in Mechanics”. St. Petersburg, Russia. 2005. Current book. Vol. 2.

7. Zhilin P.A. Applied mechanics. Theory of thin elastic rods. Tutorial book. St. Petersburg State Polytechnical University. 2006. 98 p. (In Russian). To be published.

The development of mathematical methods

An approach [1] is suggested (1995), which allows to analyse the stability of motion in the presence of spinor motions described by means of rotation (turn) tensor. The problem is that the rotation tensors are not elements of a linear space (unlike the displacement vectors). Thus the equations in variations have to be written down as a chain of equations, whose right parts depend on the previous variations in a nonlinear way. However, the obtained chain of equations allows for the exact separation of variables, i.e. the separation of the time variable.

1. Zhilin P.A. Spin motions and stability of equilibrium configurations of thin elastic rods // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1995. N 458. P. 56–73. (In Russian).

Dynamics of rigid bodies

It was the first time when the dynamics of rigid bodies was formulated in terms of the direct tensor calculus. The new mathematical technique is developed for the description of spinor motions. This technique is based on the use of the rotation (turn) tensor and related concepts. The new results in the dynamics of rigid bodies are mostly presented in the following works:

1. Zhilin P.A. Theoretical mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2001. 146 p. (In Russian).

2. Zhilin P.A. Vectors and second-rank tensors in three-dimensional space. St. Petersburg: Nestor. 2001. 276 p.

3. Zhilin P.A. Theoretical mechanics. Fundamental laws of mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2003. 340 p. (In Russian).

Development of mathematical methods

The general investigation of the rotation (turn) tensor is given (1992) in works [1, 7, 8], where a new proof of the kinematic equation of Euler is obtained. The old correct proof of the kinematic equation one could find in works by L. Euler and in old text-books on theoretical mechanics, but it was very tedious. In a well-known course by T. Levi-Civitta and U. Amaldi (1922) a new compact proof was suggested, but it was erroneous. Later this proof was widely distributed and repeated in almost all modern courses on theoretical mechanics, with exception of the book by G.K. Suslov. In work [1] the proof of a new theorem on the composition of angular velocities, different from those cited in traditional text-books, is proposed.

The new equation [1, 4–8] is obtained (1992), relating the left angular velocity with the derivative of the rotation vector. This equation is necessary to define the concept of a potential torque. Apart from that, it is very useful when solving numerically the problems of dynamics of rigid bodies, since then there is no need to introduce neither systems of angles, nor systems of parameters of the Klein-Hamilton type.

A new theorem [2, 3, 7, 8] on the representation of the rotation (turn) tensor in the form of a composition of turns about arbitrary fixed axes, is proved (1995). All previously known representations of the rotation (turn) tensors, (or, saying more precisely, of its matrix analogues) via Euler angles, Brayant angles, plane angles, ship angles etc., are particular cases of a general theorem, whose role, however, is not only a simple generalisation of these cases. The most important thing is that making a traditional choice of any system of angles, does not matter which one, we choose previously the axes. We describe the (unknown) rotation of a body under consideration in terms of turns about these axes. If this choice is made in an ineffectual way, and if it is difficult to make an appropriate choice, the chances to integrate or even to analyse qualitatively the resulting system of equations are very poor. Moreover, even in those cases when it is possible to integrate the system, often the obtained solution is not of big practical use, since this solution will contain poles or indeterminacy of the type zero divided by zero. As a result, the numerical solution, obtained with the help of computers, already after the first pole or indeterminacy becomes very distorted. The advantage and the purpose of the theorem under discussion is the fact that it allows to consider the axes of rotation as principal variables and to determine them in the process of the problem solution. As a result, one can obtain the simplest (among all possible forms) solutions.

An approach [4–6] is proposed (1997), which allows to analyse the stability of motion in the presence of spinor rotations described by the turn tensor. The method of perturbations for the group of proper orthogonal tensors is developed.

1. Zhilin P.A. The turn-tensor in kinematics of a rigid body // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1992. N 443. P. 100–121. (In Russian).

2. Zhilin P.A. A New Approach to the Analysis of Euler-Poinsot problem // ZAMM. Z. angew. Math. Mech. **75**. (1995) S 1. P. 133–134.

3. Zhilin P.A. A New Approach to the Analysis of Free Rotations of Rigid Bodies // ZAMM. Z. angew. Math. Mech. **76**. (1996) N 4. P. 187–204.

4. Zhilin P.A. Dynamics and stability of equilibrium positions of a rigid body on an elastic foundation //Proc. of XXIV Summer School - Seminar “Nonlinear Oscillations in Mechanical Systems”. St. Petersburg. 1997. P. 90–122. (In Russian).

5. Zhilin P.A. A General Model of Rigid Body Oscillator // Proc. of the XXV-XXIV Summer Schools "Nonlinear Oscillations in Mechanical Systems". Vol. 1. St. Petersburg. 1998. P. 288–314.

6. Zhilin P.A. Rigid body oscillator: a general model and some results // Acta Mechanica. Vol. 142. (2000) P. 169–193.

7. Zhilin P.A. Vectors and second-rank tensors in three-dimensional space. St. Petersburg: Nestor. 2001. 276 p.

8. Zhilin P.A. Theoretical mechanics. Fundamental laws of mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2003. 340 p. (In Russian).

New solutions of classical problems

A new solution [1, 2] is obtained (1995) for the classical problem of the free rotation of a rigid body about a fixed centre of mass (case of Euler). It is shown that for each inertia tensor all the domain of initial values is divided in two subdomains. It is known that there is no such a system of parameters, which would allow to cover all the domain of initial values by unique map without poles. This fact is confirmed in the work [2], where in each subdomain and at the boundary between them the body rotates about different axes, depending only on the initial values. Stable rotations of the body correspond to the interior points of the subdomains mentioned above, and unstable rotations — to the boundary points. When constructing the solution, the theorem on the representation of the rotation (turn) tensor, described above, plays an essential role. Finally, all characteristics to be found can be expressed via one function, determined by a rapidly convergent series of a quite simple form. For this reason, no problem appears in simulations. The propriety of the determination of axes, about which the body rotates, manifests in the fact that the velocities of precession and proper rotation have a constant sign. Remind that in previously known solutions only the sign of the precession velocity is constant, i.e. in these solutions only one axe of turns is correctly guessed. It follows from the solution [2], that formally stable solutions, however, may be unstable in practice, if a certain parameter is small enough (zero value of the parameter corresponds to the boundary between subdomains). In this case the body may jump from one stable rotational regime to another one under action of arbitrarily small and short loads (a percussion with a small meteorite).

A new solution [3, 4] for the classical problem of the rotation of a rigid body with transversally isotropic inertia tensor is obtained (1996, 2003) in a homogeneous gravity field (case of Lagrange). The solution of this problem from the formal mathematical point of view is known very long ago, and one can find it in many monographs and text-books. However, it is difficult to make a clear physical interpretation of this solution, and some simple types of motion are described by it in an unjustifiably sophisticated way. In the case of a rapidly rotating gyroscope there was obtained practically an exact solution in elementary functions. It was shown [4] that the expression for the precession velocity, found using the elementary theory of gyroscopes, gives an error in the principal term.

It is found (2003), in the frame of the dynamics of rigid bodies, the explanation of the fact that the velocity of the rotation of the Earth is not constant, and the axe of the

Earth is slightly oscillating [5]. Usually this fact is explained by the argument that one cannot consider the Earth as an absolutely rigid body. However, if the direction of the dynamic spin slightly differs from the direction of the earth axe, the earth axe will make precession about the vector of the dynamic spin, and, consequently, the angle between the axe of the Earth and the plane of ecliptics will slightly change. In this case the alternation of day and night on the Earth will be determined not by the proper rotation of the Earth about its axe, but by the precession of the axe.

1. Zhilin P.A. A New Approach to the Analysis of Euler-Poinsot problem // ZAMM. Z. angew. Math. Mech. **75**. (1995) S 1. P. 133–134.

2. Zhilin P.A. A New Approach to the Analysis of Free Rotations of Rigid Bodies // ZAMM. Z. angew. Math. Mech. **76**. (1996) N 4. P. 187–204.

3. Zhilin P.A. Rotations of Rigid Body with Small Angles of Nutation // ZAMM. Z. angew. Math. Mech. **76**. (1996) S 2. P. 711–712.

4. Zhilin P.A. Rotation of a rigid body with a fixed point: the Lagrange case //Lecture at XXXI Summer School - Conference "Advanced Problems in Mechanics". St. Petersburg, Russia. 2003. Current book. Vol. 1. (In Russian).

5. Zhilin P.A. Theoretical mechanics. Fundamental laws of mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2003. 340 p. (In Russian).

New models in the frame of the dynamics of rigid bodies

We know the role which is played by a usual oscillator in the Newtonian mechanics. In the Eulerian mechanics, the analogous role is played by a rigid body on an elastic foundation. This system can be named *a rigid body oscillator*. The last one is necessary when constructing the dynamics of multipolar media, but in its general case it is not investigated neither even described in the literature. Of course, its particular cases were considered, for instance, in the analysis of the nuclear magnetic resonance, and also in many applied works, but for infinitesimal angles of rotation. A new statement of the problem of the dynamics of a rigid body on a nonlinear elastic foundation [1, 3, 6] is proposed (1997). The general definition of the potential torque is introduced. Some examples of problem solutions are given.

For the first time (1997) the mathematical statement for the problem of a two-rotor gyrostate on an elastic foundation is given [2, 4, 5]. The elastic foundation is determined by setting of the strain energy as a scalar function of the rotation vector. Finally, the problem is reduced to the integration of a system of nonlinear differential equations having a simple structure but a complex nonlinearity. The difference of these equations from those traditionally used in the dynamics of rigid bodies is that when writing them down it is not necessary to introduce any artificial parameters of the type of Eulerian angles or Cayley-Hamilton parameters. The solutions of concrete problems are considered. A new method of integration of basic equations is described in application to a particular case. The solutions is obtained in quadratures for the isotropic nonlinear elastic foundation.

The model of a rigid body is generalised (2003) for the case of a body consisting not of the mass points, but of the point-bodies of general kind [7]. There was considered a model of a quasi-rigid body, consisting of the rotating particles, with distances between them remaining constant in the process of motion.

1. Zhilin P.A. Dynamics and stability of equilibrium positions of a rigid body on an elastic foundation //Proc. of XXIV Summer School - Seminar "Nonlinear Oscillations in Mechanical Systems". St. Petersburg. 1997. P. 90–122. (In Russian).

2. Zhilin P.A., Sorokin S.A. Multi-rotor gyrostat on a nonlinear elastic foundation // IPME RAS. Preprint N 140. 1997. 83 p. (In Russian).

3. Zhilin P.A. A General Model of Rigid Body Oscillator // Proc. of the XXV-XXIV Summer Schools "Nonlinear Oscillations in Mechanical Systems". Vol. 1. St. Petersburg. 1998. P. 288–314.

4. Zhilin P.A., Sorokin S.A. The Motion of Gyrostat on Nonlinear Elastic Foundation // ZAMM. Z. Angew. Math. Mech. **78**. (1998) S 2. P. 837–838.

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6. Zhilin P.A. Rigid body oscillator: a general model and some results // Acta Mechanica. Vol. 142. (2000) P. 169–193.

7. Zhilin P.A. Theoretical mechanics. Fundamental laws of mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2003. 340 p. (In Russian).

Dynamics of a rigid body on an inertial elastic foundation

The problems of construction of high-speed centrifuges, with rotational velocities 120 000 − 200 000 revolutions per minute, required the development of more sophisticated mechanical models. As such a model it is chosen a rigid body on an elastic foundation. The parameters of the rotor and of the elastic foundation do not allow to consider the elastic foundation as inertialess. There was proposed (1995) a method [1, 2], allowing to reduce the problem to the solution of a relatively simple integro-differential equation.

1. Zhilin P.A., Tovstik T.P. Rotation of a rigid body based on an inertial rod // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1995. N 458. P. 78–83. (In Russian).

2. Ivanova E.A., Zhilin P.A. Non-stationary regime of the motion of a rigid body on an elastic plate // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 357–363.

The Coulomb law of friction and paradoxes of Painlevé

The application of the Coulomb law has its own specifics related to the non-uniqueness of the solution of the dynamics problems. It was shown (1993), that the Painlevé paradoxes appear because of a priori assumptions on the character of motion and the character of the forces needed to induce this motion. The correct statement of the problem requires either to determine the forces by the given motion, or to determine the motion by the given forces [1, 2].

1. Zhilin P.A., Zhilina O.P. On the Coulomb's laws of friction and the Painlevé paradoxes // Mechanics and Control. Proc. of St. Petersburg State Technical University. 1993. N 446. P. 52–81. (In Russian).

2. Wiercigroch M., Zhilin P.A. On the Painlevé Paradoxes // Proc. of the XXVII Summer School "Nonlinear Oscillations in Mechanical Systems". St. Petersburg. 2000. P. 1–22.

The fundamental laws of mechanics

There were suggested (1994) the formulations of basic principles and laws of the Eulerian mechanics [1–5] with an explicit introduction of spinor motions. All the laws are formulated for the open bodies, i.e. bodies of a variable content, which appears to be extremely important when describing the interaction of macrobodies with electromagnetic fields. Apart from that, in these formulations the concept of a body itself is also changed, and now the body may contain not only particles, but also the fields. Namely, the latter ones make necessary to consider bodies of variable content. The importance of spinor motions, in particular, is determined by the fact that the true magnetism can be defined only via the spinor motions, contrary to the induced magnetism, caused by Foucault (eddy) currents, i.e. by translational motions.

A new basic object — point-body [1–5] is introduced into consideration (1994). It is assumed that the point-body occupies zero volume, and its motion is described completely by means of its radius-vector and its rotation (turn) tensor. It is postulated that the kinetic energy of a point-body is a quadratic form of its translational and angular velocities, and its momentum and proper kinetic moment (dynamic spin) are defined as partial derivatives of the kinetic energy with respect to the vector of translational velocity and the vector of angular velocity, respectively. It was considered (2003) the model of a point-body [5], whose structure is determined by three parameters: mass, inertia moment, and an additional parameter q, conventionally named *charge*, which never appeared in particles used in classical mechanics. It is shown that the motion of this particle by inertia in a void space has a spiral trajectory, and for some initial conditions — a circular trajectory. Thus it is shown that in an inertial frame reference the motion of an isolated particle (point-body) by inertia has not to follow necessarily a linear path.

There was developed (1994) a concept of actions [1–5]. This concept is based on an axiom which supplements the Galileo's Principle of Inertia, generalising it to the bodies of general kind. This axiom states that in an inertial system of reference an isolated closed body moves in such a way that its momentum and kinetic moment remain invariable. Further, the forces and torques are introduced into consideration, and the force acting upon a closed body is defined as a cause for the change of the momentum of this body, and the torque, acting upon a closed body — as a cause of the change of the kinetic moment. The couple of vectors — force vector and the couple vector — are called *action*.

The concept of the internal energy of a body, consisting of point-bodies of general kind [1–5], was developed (1994); the axioms for the internal energy to be satisfied are formulated. The principally new idea is to distinguish the additivity by mass and additivity by bodies.

The kinetic energy of a body is additive by its mass. At the same time, the internal energy of a body is additive by sub-bodies of which the body under consideration consists of, but, generally speaking, it is not an additive function of mass. In the Cayley problem, the paradox, related to the loss of energy, is resolved [5].

Basic concepts of thermodynamics [4, 5]: internal energy, temperature, and entropy are introduced (2002) on elementary examples of mechanics of discrete systems. The definition of the temperature concepts and entropy are given by means of purely mechanical arguments, based on the use of a special mathematical formulation of the energy balance.

1. Zhilin P.A. Main structures and laws of rational mechanics // Proc. of the 1st Soviet Union Meeting for Heads of Departments of Theoretical Mechanics. St. Petersburg: VIKI. 1994. P. 23–45. (In Russian).

2. Zhilin P.A. Basic concepts and fundamental laws of rational mechanics // Proc. of XXII Summer School - Seminar “Nonlinear Oscillations in Mechanical Systems”. St. Petersburg. 1995. P. 10–36. (In Russian).

3. Zhilin P.A. Theoretical mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2001. 146 p. (In Russian).

4. Zhilin P.A. Basic postulates of the Eulerian mechanics // Proc. of XXIX Summer School - Conference “Advanced Problems in Mechanics”. St. Petersburg. 2002. P. 641–675. (In Russian)

5. Zhilin P.A. Theoretical mechanics. Fundamental laws of mechanics. Tutorial book. St. Petersburg State Polytechnical University. 2003. 340 p. (In Russian).

Electrodynamics

It is shown [1, 2], that Maxwell equations are invariant with respect to the Galilean transformation, i.e. the principle of relativity by Galileo is valid for them (we distinguish transformations of frames of reference and of co-ordinate systems). The complete group of linear transformations, with respect to which the Maxwell equations are covariant, is found, and it is demonstrated that Lorentz transformations present quite a particular case of the complete group.

The role, which electromechanical analogies play in the analytical mechanics of mass points, is well-known. For the electrodynamic equations, such analogies in the modern theoretical physics are not only unknown, but are even denied. In work [3], mathematically rigorous mechanical interpretation of the Maxwell equations is given, and it is shown that they are completely identical to the equations of oscillations of a non-compressible elastic medium. Thus it follows that in the Maxwell equations there is an infinite velocity of the propagation of extension waves, which is in the explicit contradiction with special relativity theory. In other words, electrodynamics and special relativity theory are incompatible. These analogies were established by Maxwell himself for the absence of charges, and in [3] they are proved for the general case.

The modified Maxwell equations are proposed [3–5]. In the modified theory, all the waves propagate with finite velocities, one of them has to be greater than the light velocity in vacuum. If this to consider the limit case, when this velocity tends to the infinity, the modified equations give the Maxwell equations in the limit. The waves with the "superlight" velocity are longitudinal. One cannot eliminate the possibility that these waves describe the phenomenon of radiation propagating with the velocity greater than the light velocity, which is claimed to be experimentally observed by some astronomers.

It is established [3–5] that in terms of this theory electrostatic states present hyperlight waves and are realised far from the wave front.

It is shown [3], that neither classical, nor modified Maxwell equations cannot describe correctly the interaction between the electrons and the nucleus of the atom. The way to solve this problem is indicated.

It is shown [6], that the mathematical description of an elastic continuum of two-spin particles of a special type is reduced to the classical Maxwell equations. The mechanical analogy proposed above allows to state unambigously that the vector of electric field is axial, and the vector of magnetic field is polar.

1. Zhilin P.A. Galileo's equivalence principle and Maxwell's equations. // St. Petersburg State Technical University. 1993. 40 p. (In Russian).

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3. Zhilin P.A. Reality and mechanics // Proc. of XXIII Summer School - Seminar "Nonlinear Oscillations in Mechanical Systems". St. Petersburg. 1996. P. 6–49. (In Russian).

4. Zhilin P.A. Classical and Modified Electrodynamics // Proc. of Int. Conf. "New Ideas in Natural Sciences". St. Petersburg, Russia. June 17–22, 1996. Part I – Physics. P. 73–82.

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Quantum mechanics

At the end of the XIX century Lord Kelvin described the structure of an aether responsible, in his opinion, for the true (non-induced) magnetism, consisting of rotating particles. A kind of specific Kelvin medium (aether) is considered: the particles of this medium cannot perform translational motion, but have spinor motions. Lord Kelvin could not write the mathematical equations of such motion, because the formulation of the rotation tensor, a carrier of a spinor motion, was not discovered at the time. In work [1, 2] basic equations of this particular Kelvin medium are obtained, and it is shown that they present a certain combination of the equations of Klein-Gordon and Schrödinger. At small rotational velocities of particles, the equations of this Kelvin medium are reduced to the equations of Klein-Gordon, and at large velocities — to the Schrödinger equation. It is very significant that both equations lie in the basis of quantum mechanics.

1. Zhilin P.A. Reality and mechanics // Proc. of XXIII Summer School - Seminar "Nonlinear Oscillations in Mechanical Systems". St. Petersburg. 1996. P. 6–49. (In Russian).

2. Zhilin P.A. Classical and Modified Electrodynamics // Proc. of Int. Conf. "New Ideas in Natural Sciences". St. Petersburg, Russia. June 17–22, 1996. Part I – Physics. P. 73–82.

General theory of inelastic media

A general approach [1–6] for the construction of the theory of inelastic media is proposed (2001–2005). The main attention is given to the clear introduction of basic concepts: strain measures, internal energy, temperature, and chemical potential. Polar and non-polar media are considered. The originality of the suggested approach is in the following. The spatial description is used. The fundamental laws are formulated for the open systems. A new handling of the equation of the balance of energy is given, where the entropy and the chemical potential are introduced by means of purely mechanical arguments. The internal energy is given in a form, at the same time applicable for gaseous, liquid, and solid bodies. Phase transitions in the medium are described without introducing any supplementary conditions; solid-solid phase transition can also be described in these terms. The materials under consideration have a finite tensile strength; this means that the constitutive equations satisfy to the condition of the strong ellipticity.

1. Zhilin P.A. Basic equations of the theory of non-elastic media // Proc. of the XXVIII Summer School "Actual Problems in Mechanics". St. Petersburg. 2001. P. 14–58. (In Russian).

2. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

3. Altenbach H., Naumenko K., Zhilin P. A micro-polar theory for binary media with application to phase-transitional flow of fiber suspensions // Continuum Mechanics and Thermodynamics. 2003. Vol. 15. N 6. P. 539–570.

4. Altenbach H., Naumenko K., Zhilin P.A. A micro-polar theory for binary media with application to flow of fiber suspensions // Proc. of XXX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2003. P. 39–62.

5. Zhilin P.A. Mathematical theory of non-elastic media // Uspehi mechaniki (Advances in mechanics). Vol. 2. N 4. 2003. P. 3–36. (In RUssian.)

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Spatial description of the kinematics of continuum

When constructing the general theory of inelastic media there was used (2001) so called *spatial description* [1–4], where a certain fixed domain of a frame of reference contains different medium particles in different moments of time. Due to the use of the spatial description there was constructed a theory, where the concept of the smooth differential manifold is not used. Before such theories were developed only for fluids. For the first time such a theory is built for solids, where the stress deviator is non-zero. For the first time, the spatial description is applied to a medium consisting of particles with rotational degrees of freedom. A new definition of a material derivative, containing only objective operators, is given. This definition, including when using a moving co-ordinate system, does not contradict to the Galileo's Principle of Inertia [2]. It is shown that for the spatial description one can apply standard methods of the introduction of the stress tensor and other similar quantities [1]. The dynamic equations of the medium obtained basing upon the fundamental laws, formulated for the open systems. An error, which presents in the literature, appearing when integrating the differential equation expressing the law of conservation of particles, is eliminated.

1. Zhilin P.A. Basic equations of the theory of non-elastic media // Proc. of the XXVIII Summer School "Actual Problems in Mechanics". St. Petersburg. 2001. P. 14–58. (In Russian).

2. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

3. Zhilin P.A. Mathematical theory of non-elastic media // Uspehi mechaniki (Advances in mechanics). Vol. 2. N 4. 2003. P. 3–36. (In Russian.)

4. Zhilin P.A. On the general theory of non-elastic media // Mechanics of materials and strength of constructions. Proc. of St. Petersburg State Polytechnical University. N 489. 2004. P. 8–27. (In Russian).

Theory of strains

Usually in the nonlinear theory of elasticity the theory of deformations is based only on geometrical reasons, thus a lot of different strain tensors are considered. It is usually assumed that all of these tensors can be used with identical success. However, this is not correct. It is shown (2001), that the dissipative inequality imposes such restrictions on free energy [1, 2], which at use of Almansi measure of strain appear equivalent to the statement, that the considered material is isotropic. In other words it is shown, that for anisotropic materials free energy cannot be a function of Almansi measure of strain. The definition of the strain measure is given on the base of the equation of balance of energy and the dissipative inequality. It is shown, that according to the given definition the strain measure should be an unimodular tensor.

1. Zhilin P.A. Basic equations of the theory of non-elastic media // Proc. of the XXVIII Summer School "Actual Problems in Mechanics". St. Petersburg. 2001. P. 14–58. (In Russian).

2. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

Equation of mass balance and equation of particles balance

Two independent functions of state are introduced: density of particles and mass density (2002) [1–3]. Such division is important, for example, when the material tends to a fragmentation, as in this case the weight is preserved, but the number of particles changes. Permeability of bodies is determined by the density of particles, and internal interactions are connected with the mass density. Introduction of the function of distribution of particles, in essence, removes the border between discrete and continuous media. Two independent equations are formulated: the equation of mass balance and the equation of balance of particles. A function determining the speed of new particles creation appears in the equation of particles balance; this function in its physical sense can be identified with the chemical potential. The equation of energy balance also contains terms which describe formation of new particles or fragmentation of original particles.

1. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

2. Zhilin P.A. Mathematical theory of non-elastic media // Uspehi mechaniki (Advances in mechanics). Vol. 2. N 4. 2003. P. 3–36. (In Russian.)

3. Zhilin P.A. On the general theory of non-elastic media // Mechanics of materials and strength of constructions. Proc. of St. Petersburg State Polytechnical University. N 489. 2004. P. 8–27. (In Russian).

Temperature, entropy and chemical potential

Characteristics of state, corresponding to temperature, entropy, and chemical potential are obtained [1–4] from pure mechanical reasons, by means of special mathematical formulation of the energy balance equation (2001), obtained by separation of the stress tensors in elastic and dissipative components. The second law of thermodynamics gives additional limitations for the introduced characteristics, and this completes their formal definition. The reduced equation of energy balance is obtained in the terms of free energy. The main purpose of this equation is to determine the arguments on which the free energy depends. It is shown that to define first the internal energy, and then the entropy and chemical potential, is impossible. All these quantities should be introduced simultaneously. To set the relations between the internal energy, entropy, chemical potential, pressure, etc., the reduced equation of energy balance is used. It is shown that the free energy is a function of temperature, density of particles, and strain measures, where all these arguments are independent. The Cauchy-Green's relations relating entropy, chemical potential and tensors of elastic stresses with temperature, density of particles and measures of deformation are obtained. Hence the concrete definition of the constitutive equations requires the setting of the free energy only.

The equations characterizing role of entropy and chemical potential in formation of internal energy are obtained. Constitutive equations for the vector of energy flux [3] are offered. In a particular case these equations give the analogue of the Fourier-Stocks law.

1. Zhilin P.A. Basic equations of the theory of non-elastic media // Proc. of the XXVIII Summer School "Actual Problems in Mechanics". St. Petersburg. 2001. P. 14–58. (In Russian).

2. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

3. Zhilin P.A. Mathematical theory of non-elastic media // Uspehi mechaniki (Advances in mechanics). Vol. 2. N 4. 2003. P. 3–36. (In Russian.)

4. Zhilin P.A. On the general theory of non-elastic media // Mechanics of materials and strength of constructions. Proc. of St. Petersburg State Polytechnical University. N 489. 2004. P. 8–27. (In Russian.)

Theory of consolidating granular media

The general theory of granular media with particles able to join (consolidate) is developed (2001) [1, 2]. The particles possess translational and rotational degrees of freedom. For isotropic material with small displacements and isothermal strains the theory of consolidating granular media is developed in a closed form [1].

It is shown that the assumption that the tensor of viscous stresses depends on velocity, leads either to failure of dissipative inequality, or to failure of hyperbolicity [1]. Hence this assumption is unacceptable. Instead of the tensor of viscous stresses, which is frequently used in literature, the antisymmetric stress tensor is introduced [1]. For this tensor the Coulomb friction law is used. For the couple stress tensor the viscous friction law is used, and this tensor is assumed to be antisymmetric.

1. Zhilin P.A. Basic equations of the theory of non-elastic media // Proc. of the XXVIII Summer School "Actual Problems in Mechanics". St. Petersburg. 2001. P. 14–58. (In Russian).

2. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

Phase Transitions and General Theory of Elasto-Plastic Bodies

The new theory of elasto-plastic bodies is developed (2002). The theory is based on the description of the nonelastic properties by the phase transitions in the materials [1–3].

The definition of the phase transition is given in the following way. Two material characteristics are related to the density of material: solid fraction, defined as multiplication of number of particles in a unit volume on the particle volume, and porosity (void fraction), defined as unit minus solid fraction. A solid has several stable states, corresponding to different values of solid fraction. Transition from one stable state to another is a typical phase transition. The constitutive equation describing the solid fraction change near the phase transition point is suggested.

The constitutive equation for elastic pressure is proposed [1]. This equation describes well not only gases and liquids, but also solids with phase transitions. The limited tensile strength is taken into account. The difference between solids and liquids mainly is in their reaction on the shape change. This reaction can be described only if the stress tensor deviator is taken into account. For the classical approach the deviator of the elastic stress tensor, which is independent of velocities by definition, is ignored for description of inelastic properties of materials. For solids this is unacceptable. One of the problems of the theory is the definition of the internal energy structure in a way that would make possible the existence of several solid phases. The constitutive equations for the stress tensor deviator are suggested [1], where the shear modulus depends on the state parameters (temperature, mass density, deformation).

1. Zhilin P.A. Phase Transitions and General Theory of Elasto-Plastic Bodies // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 36–48.

2. Zhilin P.A. Mathematical theory of non-elastic media // Uspehi mechaniki (Advances in mechanics). Vol. 2. N 4. 2003. P. 3–36. (In Russian.)

3. Zhilin P.A. On the general theory of non-elastic media // Mechanics of materials and strength of constructions. Proc. of St. Petersburg State Polytechnical University. N 489. 2004. P. 8–27. (In Russian.)

Micro-polar theory for binary media

Micro-polar theory for binary media is developed (2003) [1, 2]. The media consists from liquid drops and fibers. The liquid is assumed to be viscous and non-polar, but with nonsymmetric stress tensor. The fibers are described by nonsymmetric tensors of force and couple stresses. The forces of viscous friction are taken into account. The second law of thermodynamics is formulated in the form of two inequalities, where the components of the binary media can have different temperatures.

1. Altenbach H., Naumenko K., Zhilin P. A micro-polar theory for binary media with application to phase-transitional flow of fiber suspensions // Continuum Mechanics and Thermodynamics. 2003. Vol. 15. N 6. P. 539–570.

2. Altenbach H., Naumenko K., Zhilin P.A. A micro-polar theory for binary media with application to flow of fiber suspensions // Proc. of XXX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2003. P. 39–62.

Development of mathematical methods

The theory of symmetry for tensor quantities is developed. The new definition for tensor invariants is given (2005) [1, 2]. This definition coincides with the traditional one only for the Euclidean tensors. It is shown that any invariant can be obtained as a solution of a differential equation of the first order. The number of independent solutions of this equation determines the minimum number of invariants necessary to fix the system of tensors as a solid unit.

1. Zhilin P.A. Modified theory of symmetry for tensors and their invariants // Izvestiya VUZov. Severo-Kavkazskii region. Estestvennye nauki (Transactions of Universities. South of Russia. Natural sciences). Special issue "Nonlinear Problems of Continuum Mechanics". 2003. P 176–195. (In Russian).

2. Zhilin P.A. Symmetries and Orthogonal Invariants in Oriented Space // Proc. of XXXII Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2005. P. 470–483.

Piezoelasticity

Equations of piezoelasticity are obtained (2002–2005) [1, 2]. These equations contain as particular cases several theories, two among them are new. The proposed general theory is based on the model of micro-polar continuum. The main equations are derived from the fundamental laws of the Eulerian mechanics. These equations contain nonsymmetric tensors of stress and couple stress.

1. Kolpakov Ja. E., Zhilin P.A. Generalized continuum and linear theory of the piezoelectric materials // Proc. of XXIX Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg. 2002. P. 364–375.

2. Zhilin P.A., Kolpakov Ya.E. A micro-polar theory for piezoelectric materials // Lecture at XXXIII Summer School – Conference "Advanced Problems in Mechanics". St. Petersburg, Russia. 2005. Current book. Vol. 2.

Ferromagnetism

The theory of the nonlinear elastic Kelvin medium whose particles perform translational and rotational motion, with large displacements and rotations, and may freely rotate about their axes of symmetry, is proposed. The constitutive and dynamic equations are obtained basing upon the fundamental laws of the Eulerian mechanics. The exact analogy is established between the equations for a particular case of Kelvin medium and the equations of elastic ferromagnetic insulators in the approximation of quasimagnetostatics (1998–2001) [1–3]. It is shown that the existing theories of magnetoelastic materials did not take into account one of the couplings between magnetic and elastic subsystem, which is allowed by fundamental principles. This coupling is important for the description of magnetoacoustic resonance, and may manifest itself in nonlinear theory as well as in the linear one for the case of anisotropic materials.

1. Grekova E.F., Zhilin P.A. Ferromagnets and Kelvin's Medium: Basic Equations and Magnetoacoustic Resonance // Proc. of the XXV–XXIV Summer Schools "Nonlinear Oscillations in Mechanical Systems". Vol. 1. St. Petersburg. 1998. P. 259–281.

2. Grekova E.F., Zhilin P.A. Equations for non-linear elastic polar media and analogies: Kelvin medium, non-classical shells, and ferromagnetic insulators // Izvestiya VUZov. Severo-Kavkazskii region. Estestvennye nauki (Transactions of Universities. South of Russia. Natural sciences). Special issue "Nonlinear Problems of Continuum Mechanics". 2000. P 24–46. (In Russian).

3. Grekova E.F., Zhilin P.A. Basic equations of Kelvin's medium and analogy with ferromagnets // Journal of elasticity. Vol. 64. (2001) P. 29–70.

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