A.I. Lurie — Works on Mechanics*

Abstract

The report is devoted to contribution by A.I. Lurie in the development of mechanics in Russia. It is necessary to underline, that the scientific interests of A.I. Lurie were extremely wide and concerned to different fields of mechanics and the control process. The books and textbooks by A.I. Lurie, which were studied by hundred thousand of students, engineers and scientific workers, show high samples of scientific creativity. In 1927 the Leningrad mechanical society was founded, which has played an important role in the development of mechanics in USSR. The one of organizers of this society was A.I. Lurie. Among many achievements of the society there was organization of the issuing of well known journal "Applied Mathematics and Mechanics". A.I.Lurie was an editor of translations of many remarkable books on a mechanics. A.I. Lurie is recognized by the scientific community as the distinguished scientist - encyclopedist. A.I. Lurie was by a member of National Committee of USSR on theoretical and applied mechanics, and in 1961 he was selected by corresponding member Academy of Sciences of USSR. The name of A.I. Lurie has come in the history of mechanics in Russia for ever.

1 Introduction

A man cannot choose his birthday or birth place. However, time and habitation country significantly influence upon the making of a person and determine the character of his activity. Nevertheless, at all times and in all countries the individuals are born, who are realized as self-independent and self-sufficient creatures. Such persons play the role of "evolution catalysts" for the society, into which they are involved. The problems solved by them are never accidental but determined by the higher necessities of the society. The main feature of a realized individual is a capacity of a person not only to perceive intuitively the society higher necessities but to take them as a guide to the action. Therefore, this is impossible to make a correct evaluation of a contribution of any person into the evolution of a community (either of its certain part) if one does not realize clearly the state of this community and its necessities at the evolution stage considered. No doubt, Anatoly Isakovich Lurie had realized himself as a self-independent individual, whose versatile fruits of work we sense so distinctly. The aim of this presentation is a discussion of

*Zhilin P.A. "A.I. Lurie — Works on Mechanics" // Proceedings of XXVIII Summer School – Conference "Advanced Problems in Mechanics", St. Petersburg, Russia, 2001. P. 1–13. (In Russian)

A.I. Lurie's contribution into evolution of mechanics in Russia. A.I. Lurie had began his self-independent investigations in mechanics in 1925 all at once on graduating from the Faculty of Physics and Mechanics of Leningrad Polytechnical Institute. Think of Russia being in 1925! The previous decade resulted in an extremely hard state for Russia. The First World War, the October Socialist Revolution, and, finally, the fratricidal civil war, which is the worst and the most dangerous among all kinds of wars. All this had led to the scarcity and dissociation of the Russian brain-power to nearly complete destruction of relatively weak industry together with the total absence of finances for purchase of needed equipment. In addition, Russia was, actually, in a complete isolation from the all-world community. Consequently, development of the native industry became one of immediate tasks. Traditionally, only ship building was rather well-developed, but other fields of industry (such as mechanical engineering, power engineering, turbine construction, instrument-making and aircraft industries, etc.), they all were present in embryo. Everything mentioned above had to be built up anew. First of all, tens of thousands of skilled engineers were to be trained. It should be taken into account, that these skilled engineers had to be prepared from a relatively uneducated medium, since schools worked under abnormal conditions in 1914-1922 as well. For training a skilled engineer brain-power competent specialists and, also, text-books were needed. It cannot be said that there were no scientists in the field of mechanics in Russia. Suffice it to recall such first-class scientists as N.E. Zhukovsky, I.G. Bubnov, I.V. Meschersky, A.A. Fridman, A.N. Krylov, P.F. Papkovich, E.L. Nicolai, and many others. However, they were extremely few in number for such a vast country as Russia. As for text-books on mechanics for universities, they were actually absent. Just then, the generation of Russian scientists, to which A.I. Lurie belonged, had to start the work. Creative work of the above-mentioned scientists received a high appraisal by the launching of the first in the world artifical satellite on October 4th, 1957, along with the fact that to 1960 the technical education in Russia was recognized as one of the best in the world by the international community.

On graduating from the Polytechnical Institute A.I. Lurie hold the post of a lecturer at the chair "Theoretical Mechanics" of the institute. Hereafter, A.I. Lurie began his persistent research work. It is necessary to emphasize that A.I. Lurie was utterly interested in various fields of mechanics and of control theory. It is accounted for by the fact that A.I. Lurie was tightly concerned with organizations engaged in development and production of new technique. Among the organizations, Leningradskii Metallicheskii Zavod (a Leningrad Metal Plant), Osoboe Tekhnicheskoe Byuro (the Special Technical Department), and Osoboe Konstruktorskoe Byuro (the Special Constructor Department) to be pointed out in the first place. As it is known, creation of a new technique is accompanied by numerous problems associated with mechanics and the control theory. Over the post-war years, contacts of A.I. Lurie with industrial organizations were essentially widened. Multiform demands of practice made the scientist to perform his investigations simultaneously in various directions. Therefore, in describing the works of A.I. Lurie on mechanics we ought to divide the works into separate groups and to break the chronological succession. As for investigation on the control theory, into which A.I. Lurie made a valuable contribution, they represent the subject of a separate consideration.

2 At the source of the Leningrad School of Mechanics

A.I. Lurie was not only the eminent Scientist, but a striking Teacher as well. He left hundreds of disciples in mechanics, many of them became world-known scientists. Monographs and text-books by A.I. Lurie, by which hundreds of thousands of future engineers studied mechanics, continue to remain brilliant examples of scientific creative work. The scientific style of A.I. Lurie was remarkably rigorous and clear, without any pseudo-scientific excesses. The scientist gave a lot of efforts to the development of a mathematical technique that allowed to solve the problems in the most effective and clear way. In particular, A.I. Lurie was a staunch devotee to the direct tensor calculus, and he made a remarkable contribution into development and introduction of this new technique.

In 1927, Leningrad's Mechanical Society had been established, which played a great role in development of mechanics in the USSR. Prof. E.L. Nicolai was the organizer and the permanent President of the Society, whereas A.I. Lurie was its Scientific Secretary. Among many achievements of the Society, one should point out the organization of edition of the first in the USSR specialized journal on mechanics and applied mathematics. Initially, beginning from 1929, the title of the journal was "Vestnik Mekhaniki i Prikladnoy Matematiki" ("News of Mechanics and Applied Mathematics"). In 1933, the journal was transformed into an all-union periodic edition "Applied mathematics and mechanics" ("Prikladnaya Matematika i Mekhanika"). Up to 1937, when edition of the journal was transferred to Moscow and referred to the Institute of Problems in Mechanics, Russian Academy of Sciences USSR, E.L. Nicolai was appointed as the Editor-in-Chief, whereas A.I. Lurie worked as the Executive Editor.

As it was mentioned above, at the time the special literature on mechanics was, actually, absent in Russia. One had to study mechanics by English, German, and French original publications, which was possible for A.I. Lurie but not for many those, to whom the knowledge of mechanics was necessary. Therefore, there were severe need in edition of scientific literature translations. A.I. Lurie was actively involved in this important work. In particular, a lot of translations of remarkable monographs were edited by the scientist, for instance, such as E. Trefftz, Mathematical theory of elasticity (1934); Hekkeler, Statics of elastic body (1934); P. Pfeiffer, Oscillations of Elastic Bodies (1934); Analytical mechanics by Lagrange (1938); C. Truesdell, First Course in Rational Continuum Mechanics (1975) and so on. Note that in thirties, the translations of foreign editions played a significant role in training the engineers in the USSR.

Scientific merits of A.I. Lurie are universally recognized. The world scientific community knows him as a prominent scientist-encyclopedist. A.I. Lurie was a member of the National Committee of the USSR on Theoretical and Applied Mechanics, and in 1961 Prof. A.I. Lurie was elected as a corresponding member of Academy of Sciences of the USSR. V.V. Novozhilov, Academician of RAS of the USSR, stated that A.I. Lurie was attributed to those selected scholars, for whom the highest scientific titles were their names.

3 Operational calculus

Early works of A.I. Lurie were devoted to hydrodynamics of viscous liquids. This was the subject of his thesis defended in 1929. Generally speaking, no theses were defended at that time. Nevertheless, A.I. Lurie wrote his thesis and it was discussed at the meeting of Academic Council. V.A. Fok and A.A. Satkevich, well-known professors, were the referees. The thesis was defended successfully, and a positive decision was sent to the Council Record Office. In the USSR the advanced degrees were re-established only in 1933, and just at that time A.I. Lurie gained a honorary doctorate. Note that earlier he had attained a Professor title already. Although the works on hydrodynamics of viscous liquids are not among most important achievements of A.I. Lurie, nevertheless, the scholar pioneered in applying an approach based on operational calculus, which was new for this field of mechanics. The approach was approved by Academician V.A. Fok, and he advised A.I. Lurie to continue investigations in this direction. These investigations resulted in publication of the paper “On the theory of the sets of linear differential equations with constant coefficients” (Trudy Leningradskogo Industrialnogo instituta – Transactions of Leningrad Industrial Institute, 1937, No. 6, pp. 31–36) and of a monograph “Operational calculus” (Moscow–Leningrad: ONTI, 1938, in Russian). Later, these investigations were developed and resulted in creation of A.I. Lurie’s symbolic method discussed below in section devoted to the elasticity theory.

The idea of operational calculus was proposed by Oliver Heaviside in 1893. Further this idea was extended in works of T.J. Bromwich, E.P. Adams, H. Jeffreys and some other western scientists. Usually, operational methods were applied to the calculation of electric circuits. Meanwhile, at the end of thirties they did not have a wide application in mathematical physics. As H. Jeffreys pointed out, this happened since there were some obscurities in the basic theory and there were no systematic description of operational methods. For the first time, the systematic theory of operational methods was described in the book *Harold Jeffreys. Operational methods in mathematical physics. London, Cambridgde, 1927.* The second edition of the book was published in 1931. We see that in 1930 the operational calculus became rather popular, mainly, in England. Therefore it would not be correct to speak about A.I. Lurie’s contribution into operational calculus. The scientist is worthy in another matter. Firstly, the West is the West, whereas Russia of thirties was a country, where it was a great problem to become acquainted with achievements of foreign scientists. Secondly, abstract ideas, let them even be rather perspective, were not completely appropriate for the technical education in Russia of that time. There was a need in convincing applications of the ideas to the concrete technical problems. Just that was made by A.I. Lurie. In particular, two well-known problems were considered in his paper of 1937 mentioned above. The first one was the problem of a body in a flow of viscous liquid. On the basis of operational calculus, there was derived a solution, merely in a few lines, which had been obtained by L.S. Leybenson in 1935 by another method. The second example was a derivation of the general solution for equations of statics in the linear elasticity. This solution, without derivation, was published by B.G. Galerkin in 1930 in Doklady AN SSSR. Earlier B.G. Galerkin made a presentation on this subject at the Meeting of Leningradian Mechanical Society. During the presentation he only wrote the formulae of the solution on the blackboard and suggested to the colleagues to check that the formulae are the solution of equations of statics and allow to satisfy arbitrary boundary conditions. For the first time, A.I. Lurie gave in his paper the complete derivation of Galerkin’s solution. In the same paper, he obtains by this method the solution of Lamé dynamic equations, which gives Galerkin’s solution as a particular case. In the monograph “Operational calculus” one can find a lot of solved problems which are of independent significance and pronounced technical trend. Owing to the fact, the monograph became a manual for engineers engaged in different organizations related to calculations. The same destiny waited for majority of another works by A.I. Lurie.

In conclusion, we note one more work by A.I. Lurie. Towards the end of the thirties, at Leningradskii Metallicheskii Zavod (Leningrad Metal Plant), in the course on construction of powerful vapor turbines, a phenomenon, new for that time, namely, self-excitation of severe vibrations in high-pressure pipelines, was discovered. Later, the phenomenon was named as the hydrodynamical shock. The vibration occurred to be so active that the walls of a huge workshop stated to shake. A.I. Lurie was involved into solving of problem. He constructed a mathematical model and made the calculations with in collaboration with his coworker A.I. Chekmarev. The observed phenomenon of self-excitation of vibrations in a pipeline was completely borne out by the above calculations. This seemed to be the first solution of a problem of such type. Far later, similar calculations were performed by A.I. Lurie's students and colleagues, namely, V.A. Palmov, A.A. Pervozvansky, V.A. Pupyrev under the guidance of A.I. Lurie for pipelines of another types. For the solution of the problem, the operational calculus was used as well. Difficulty arose with formulation of the criteria of stability. Usual criteria (of Gurvitz, Mikhailov type etc.) were not applicable, since critical values had to be found from the transcendental equation. This problem is not solved yet for the time being for the general case. However, the numerical calculations were completely performed.

4 Analytical mechanics

It was mentioned that just after graduating from the institute A.I. Lurie began to teach theoretical mechanics. At that time a well-known scientist I.V. Mescherskiy hold the Chair of Theoretical Mechanics, who, in addition, was the author of the unique problem book on theoretical mechanics, which, up to date, had gone into 38 editions and it had been translated into many foreign languages. By the way, just I.V. Mescherski pioneered in the introduction of the exercises as a form of education. One should take into account, that at that time in the world there was no such a subject "theoretical mechanics" as an element in the technical education. There existed the subject "analytical mechanics", which was studied at the mathematical faculties of the universities. One can mention famous monographs "Analytical dynamics" by E.T. Whittaker and "Theoretical mechanics" by P. Appel. There were also some other textbooks, but they were not translated into Russian. In 1922 in Russia the lithographic lecture notes on the analytical mechanics by N.V. Roze, a Professor of Leningrad State University, were published. All these monographs were not very appropriate for teaching in technical institutes, from which practical engineers were graduating. Thus there was a vital necessity in such a textbook on theoretical mechanics for technical institutes, a textbook, which could present in an understandable way all the achievements of the theory together with its practical applications. At the time being, when the development of the fundamental mechanics in Russia is on the very high leveln the West, one can hardly imagine all the hugeness of the task confronting the Russian technical education at that time. The slogan "to overtake and surpass!" was not even put on agenda there, and "to overtake!"

slogan seemed to be a far remote dream. A.I. Lurie, being a highly educated person, realized perfectly all this. The scientist not only realized the situation, but also he made every effort to change it. As a result, in 1932–1933 there was published a monograph in three volumes: L.G. Loytsianski, A.I. Lurie "Theoretical mechanics". The first and the second volumes contained the material required for technical education of an engineer, and the third volume contained more sophisticated methods of the analytical mechanics with applications to the large amount of specific problems. Later on, the contents of the first two volumes was accepted as the compulsory program for the technical institutes. All next editions of this text-book did not include the third volume. The sixth (and the last) edition of the monograph had been published in 1983, after the death of A.I. Lurie. This monograph was novel in many relations. Firstly, contrary to its western analogues, it paid a lot of attention to the technical applications. Accordingly, certain theoretical problems of the analytical mechanics, which might "frighten" practical engineers, were omitted. On the contrary, applied aspects were far more extended. Wide applicability of vectorial calculus, nearly unused at that time, gave the additional clarity to the book. A small book by L. Silberstein ("Vectorial mechanics". London: McMillan & Co., 1913) was the only book on mechanics of that time using vectorial calculus. However, the book by Silberstein was absolutely unuseful for the purposes of the technical education, and, apart from that, it used the terminology, which had not been accepted later on. The textbook by L.G. Loytsiansky and A.I. Lurie played a great role in education of Russian engineers. By the way, the theoretical mechanics was one of compulsory courses, not less than of 230 hours, in all technical institutes of Russia. This was one of main reasons of the high educational level of Russian engineers. Regrettably, beginning from the late sixties, the volume of courses on mechanics in many technical institutes has decreased steadily, and this led to the declining of the level of the graduate mechanical engineers. Since the technical education made the foundation for entire higher education in Russia, declining in the professional level of engineers led to the diminution of the IQ of the Russian population as a whole. We compare not more than two numbers characterizing the IQ of the Russian population, namely, in 1960 Russia took the second place among all the world countries with respect to this quality, whereas in 1995 our country was merely at 54th place. Of course, reduction of a role of mechanics in the technical education is not the only reason for such poor situation, but this is one of the main problems. It is out of line here to go to the detailed discussion of such a burning (and not only for Russia) question, however, this is a indisputable fact.

Now, we turn to the description of the creative work of A.I. Lurie. After the publication of the textbook on theoretical mechanics, the scientist's research interests had been concentrated in another fields of mechanics (to be discussed further), for the period of almost 20 years. This statement is not completely true, because during all these years A.I. Lurie gave courses in various areas of mechanics, including the course on analytical mechanics for students of the Faculty of Physics and Mechanics, and, naturally, the scientist continued to cogitate on the analytical mechanics problems. However, his publications of those years were devoted to other problems. At the beginning of the fifties, due to necessities in computation of motion of artificial satellites and solution of some other problems, for instance, in developing the gyroscopic systems, A.I. Lurie interests, again, turned to the analytical mechanics. As a result, in 1961, the fundamental monograph "Analytical Mechanics" was published. It should be noticed that Russia already scored big successes in the field of education at that time. Development of the fundamental mechanics in Russia gained the level of leading countries of the West. As for certain fields in mechanics, for instance, the theory of gyroscopic systems, Russia had taken the leading position. The monograph "Analytical mechanics" to the full extent supports the said above. In the monograph not only all the basic methods of analytical mechanics were presented, but their essential development was made. The monograph, as a whole, held the peculiar features of A.I. Lurie's creative scientific work, such as clarity and laconism in the presentation of a subject along with a high theoretical level and pronounced trend to the applied science. Very few scientists succeeded in performing the synthesis of such a kind. Everyone who encounters a need to solve any problem in the field of dynamics of systems with the finite number degrees of freedom, might be advised to look through, first of all, "Analytical mechanics" of A.I. Lurie. This is very probable that the reader will find the problem needed or something very much alike in this monograph. In many respects the monograph can be named as an encyclopaedia or a reference book. However, in comparison with encyclopaedia or a reference book, all the problems are discussed basing on the same foundation, along with the thorough treatment of all the details there. Of course, this leads to the apparition of new elements in many of these problems. This would take a lot of efforts to describe all these new elements, but no doubt that the reader will discover them easily himself. To illustrate this, we shall mention the description of the relative motion, and make an emphasis on the kinematics of rigid bodies, embedded by rotors (gyroscopes). Below we restrict our description to short references for a few those new elements[1], which are of theoretical significance, i.e. just they make their contribution to the bases of analytical mechanics. Here, first of all, one must mention the description of rotation of a rigid body by means of the vector of finite rotation. The vector, as such, was known long ago. Nevertheless, even in modern textbooks on physics and in some contemporary papers in the field of mechanics the possibility to describe the rotation by means of vector is denied. This fact is caused by the erroneous application of the concept of the superposition of rotations. A detailed mathematical apparatus for effective using of the finite rotation vector is developed in "Analytical mechanics". In particular, the theorem is derived which gives the expression for the finite rotation vector corresponding to the superposition of rotations via finite rotation vectors for the composing rotations. The rule of inversion for the finite rotation vectors is established. The formula giving the relation between the angular velocity vector and the time derivative of the finite rotation vector. The Darboux problem, i.e. the problem for determination of rotations by given vector of angular velocity, is formulated in terms of the finite rotation vector. The formulae giving the relation between the finite rotation vector and the Rodrigo–Hamilton and Cayley–Klein parameters, are established. The significance of the above results is concerned with the fact that they can not become outdated, i.e. once and forever, they have gone into mechanics. One more fundamental result is the following. By the end of the nineteenth century, Rayleigh introduced the concept of the dissipative function as the quadratical form of velocities. This disspative function was very useful in the analysis of nonconservative systems. Regrettably, the Rayleigh dissipative function was defined only for one class of friction, namely, for the linear viscous friction. In the monograph "Analytical mechanics" the concept of the dissipative function was generalized for an arbitrary dependence of friction forces on the velocity. One can find in the

[1]This is hardly possible to propose a lot of such new elements in mechanics

book the examples of the dissipative functions for various friction laws. In particular, the dissipative function for the Coulomb friction law is constructed. Now, the function is widely used in problems of dynamics of the systems with Coulomb friction.

5 The theory of thin elastic shells

A great number of A.I. Lurie's works is devoted to theories of thin rods, plates and shells. In this section, we concentrate our attention on the theory of shells. The theory of shells is one of the most actual directions of research in mechanics. This is caused by the following circumstances. Firstly, thin-walled constructions are widely applied in technology and civil engineering. By the way, the Nature also widely uses thin-walled elements, e.g. biological membranes, in biological systems. Secondly, in the theory of shells the general mechanics, generalized in comparison to the Newtonian mechanics, is developed in the explicit way. A.I. Lurie actively worked on the theory of shells during more than 25 years. His first paper in this field "The investigations on the theory of elastic shells" (Trudy Leningradskogo Industrialnogo Instituta — Transactions of Leningrad Industrial Institute, 1937, No. 6, pp. 37–52) had issued in 1937, and the last one "On the statical geometrical analogy of the theory of shells" was published in 1961. As a whole, A.I. Lurie had published five extensive papers and one monograph. The monograph, "Statics of the thin elastic shells" (Gostekhizdat, 1947, 252 p., in Russian), played an important role of a reliable scientific basis for practical calculations. It was the first monograph by a Russian author specialized in the theory of shells. One should mention that the theory of shells was one of the first[2] areas in mechanics of solids, where, as long ago in 1940, the post-revolutionary Russia not only had achieved the level of well-developed western countries, but even left them behind. The role of A.I. Lurie in this success can not be overemphasized, though, undoubtely, achievements of other Russian scientists, among whom A.L. Goldenweizer and V.V. Novozhilov must be mentioned, are very significant. In the first cited above work A.I. Lurie writes: "As compared to the that, hardly understandable, presentation of the subject given in chapter XXIV of the well-known work by Love[3], we, using the language of the vectorial notation, have simplified essentially all derivations". We emphasize that in this work, as well as in all his works, A.I. Lurie applies the most modern versions of the corresponding mathematical theories. In the above case it was the geometry of surfaces. In the work under discussion A.I. Lurie proposed a rigorous theory for infinitesimal deformations of surfaces for enough general case. At the same time, when deriving the equilibrium equations in displacements, A.I. Lurie applied a bit modified but nevertheless restricted method proposed by B.G. Galerkin two years before. In his work "The general theory of elastic thin shells" (PMM, 1940, IV, N 2, pp. 7–34, in Russian) A.I. Lurie already took off all the restrictions and developed the complete theory based on Kirchoff–Love hypotheses, in terms of tensor calculus. Even at present, the theory by A.I. Lurie can not be improved without abandon the Kirchoff–Love hypotheses. The monograph "Statics of thin elastic shells" is quite characteristic for all the creative work of the scientist. In all his investigations, he never forgot for whom

[2] Among other areas where the priority of Russian scientists is unquestionable, one must point out Kolosov–Muskhelishvili method for a plane elasticity problem.

[3] August Love. Mathematical theory of elasticity. Moscow, Leningrad: ONTI, 1935

his works were written. In the case of the mentioned monograph, he beared in mind numerous groups of engineers engaged in various design and constructor bureau. For this reason the tensor calculus was not used in the book. The presentation is ultimately clear and simple, but also rigorous, and is limited to examination of most usable classes of shells, mainly, by shells with rotational symmetry. In this monograph one can find a great number of solved problems, with easily used in practice design formulae. This is a well-known fact that the equations of the theory of shells are cumbersome, and their solutions are awkward and can be hardly used in engineering calculations. Taking this into account, A.I. Lurie renounced from presentation of exact solutions, which are dubious to the certain degree because of approximate character of the theory of shells itself, and he did apply asymptotical methods. As a result, he succeeded to obtain compact and easy in use formulae for computations. The above mentioned features of the monograph had made it a manual handbook for calculating engineer just after the publication. One would mistaken to believe that the monograph in question is of no other than applied significance. The results presented referred to the most important achievements of the theory. Actually, the theory of shells was inspired by vital practical necessities. Therefore, it would be useless to write down cumbrous equations and even more unmanageable solutions, which appeared often in the beginning of the XX century and were never applied anywhere. The monograph by A.I. Lurie helped the theory of shells to escape this sad destiny. The asymptotical formulae obtained in the monograph were a result of quite rigorous mathematical analysis. One should take into account that the theory of differential equations with an infinitesimal parameter in the coefficients of the highest derivatives had not been developed yet. It had appeared ten years later, and it grew up just from the problems of the theory of shells. Among concrete results discussed in the monograph, one has to point out the problem on the stress concentration in the vicinity of a hole at the surface of the cylindrical shell. The classical Kirsch problem on the stress concentration near the hole in the plane subjected to the extension, is known. The problem solved by A.I. Lurie is a far generalization of the Kirsch problem. Afterwards, this problem gave rise to the separate large part of the theory of shells. Without dwelling on other A.I. Lurie's results in this field, we note that all the six his works on the theory of shells became classical and now they are an integral part of the modern theory of shells.

6 Spatial problems of linear and nonlinear elasticity theories

A.I. Lurie devoted a great number of his scientific works to the spatial problems of the elasticity theory. We concentrate our attention on three of them, namely:

1. Spatial problems of the theory of elasticity. Gostekhizdat, 1955, 491 p.
2. Theory of elasticity. Nauka, 1970, 939 p.
3. Nonlinear theory of elasticity. Nauka, 1980, 512 p.

All the three monographs are related to one and the same area of mechanics. Meanwhile, they are not intercrossed with respect to contents. The first book concerns rigorous solutions for problems on statics of elastic bodies. The attention is mainly focused on analysis of problems for an elastic layer. Just in this monograph A.I. Lurie proposed a new method, which became a widely known as a Lurie symbolic method. This approach is a far extended generalization of operational methods, but it has also essential differences. Let us demonstrate the idea of this method using as an example the problem for an elastic layer. We put down the Lamé static equation for a layer $|z| \leqslant h$:

$$\nabla \cdot \nabla \mathbf{u}(x, y, z) + \frac{1}{1-2\nu} \nabla\nabla \cdot \mathbf{u}(x, y, z) = \mathbf{0}, \qquad |z| < h, \quad x, y \in \Omega, \tag{1}$$

where $\mathbf{u}$ is the displacement vector, and the body forces are omitted for simplicity. The nabla operator can be represented as follows

$$\nabla = \mathbf{k} \frac{d}{dz} + \boldsymbol{\Lambda}, \qquad \boldsymbol{\Lambda} = \mathbf{i} \frac{d}{dx} + \mathbf{j} \frac{d}{dy}, \qquad \mathbf{k} \cdot \boldsymbol{\Lambda} = 0. \tag{2}$$

Substituting equation (2) into equation (1), we rewrite the last one as

$$\frac{d^2}{dz^2} \left(\mathbf{E} + \frac{1}{1-2\nu} \mathbf{k}\,\mathbf{k} \right) \cdot \mathbf{u} + \frac{1}{1-2\nu} (\mathbf{k}\,\boldsymbol{\Lambda} + \boldsymbol{\Lambda}\,\mathbf{k}) \cdot \frac{d\mathbf{u}}{dz} + \\ + \boldsymbol{\Lambda} \cdot \boldsymbol{\Lambda}\,\mathbf{u} + \frac{1}{1-2\nu} \boldsymbol{\Lambda}\,\boldsymbol{\Lambda} \cdot \mathbf{u} = \mathbf{0}. \tag{3}$$

If we consider operator $\boldsymbol{\Lambda}$ in this equation as a vector, which does not depend on the variable z, then equation (3) is an ordinary differential equation with constant coefficients. Let us add "initial" conditions to equation (3)

$$z = 0: \qquad \mathbf{u} = \mathbf{f}(x, y), \quad \frac{d\mathbf{u}}{dz} = \mathbf{g}(x, y). \tag{4}$$

Thus we obtain the initial value problem (3)–(4), where variables x, y are considered as parameters. Particular solutions of the problem are sought in the form of

$$\mathbf{u} = \exp(i\lambda z)\, \mathbf{a}(x, y). \tag{5}$$

Substituting representation (5) into equation (3) we obtain a homogeneous set of equations for a vector $\mathbf{a}$

$$\left[-\lambda^2 \left(\mathbf{E} + \frac{1}{1-2\nu} \mathbf{k}\,\mathbf{k} \right) + \frac{i\lambda}{1-2\nu} (\mathbf{k}\,\boldsymbol{\Lambda} + \boldsymbol{\Lambda}\,\mathbf{k}) + \boldsymbol{\Lambda} \cdot \boldsymbol{\Lambda}\,\mathbf{E} + \frac{1}{1-2\nu} \boldsymbol{\Lambda}\,\boldsymbol{\Lambda} \right] \cdot \mathbf{a} = \mathbf{0}. \tag{6}$$

Let tensor $\mathbf{A}$ be the expression within square brackets in this equation. Nontrivial solutions for equation (6) exist if the determinant of tensor $\mathbf{A}$ equals 0. By evaluating the determinant we derive an equation to determine the characteristic values λ:

$$\det \mathbf{A} = \frac{2(1-2\nu)}{1-2\nu} \left(\lambda^2 - D^2 \right)^3 = 0, \quad \text{where } D^2 = \boldsymbol{\Lambda} \cdot \boldsymbol{\Lambda} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{7}$$

This equation has two roots $\lambda = \pm D$, each of the multiplicity 3. Having done simple calculations we obtain the following representation for a solution of the initial value problem:

$$\mathbf{u} = \mathbf{P}(z, zD) \cdot \mathbf{f}(x, y) + \mathbf{Q}(z, zD) \cdot \mathbf{g}(x, y). \tag{8}$$

Tensors **P** and **Q** are to be considered as differential operators of the infinite order. They are analytical functions of operators D^2, $D \sin zD$, $\cos zD$. To write down the explicit form for tensors $\mathbf{P}, \mathbf{Q}$ we have to represent them in terms of series that include only integer powers of operator D^2, which is the two-dimensional Laplacian. Now, we have to derive the equations to determine functions $\mathbf{f}, \mathbf{g}$. We do this using the boundary conditions at $z = \pm h$. Finally, we obtain two vector equations with two-dimensional differential operators of the infinite order. Note that series for these operators converge very fastly. Therefore, usually it is possible to take into account only a few terms of the series. For instance, if we take into account only the principal term, we obtain equations of the classical Kirchoff theory of plates. The next approximation gives us the theory of plates taking into account a transverse displacement deformation. We shall not go into additional details here. However, we emphasize that the symbolic method described above has an absolutely rigorous mathematical proof. This is easy to generalize the method for the dynamical case, and it had been performed. The symbolic method of A.I. Lurie has had wide applications in the theory of thick plates, and it was used by many authors. The method turned to be the most effective in combination with the technique of homogeneous solutions, to which A.I. Lurie made a significant contribution as well.

The monograph “Theory of elasticity”, near 650 pages, is beyond competition as for its fundamental nature and the scope of printed matter on the static problems of the elasticity. The dynamic problems and waves in elastic media are not considered in the book. The reason is not only the wish ot avoid the inevitable excessive increasement of the book size, but chiefly the fact that dynamic problems essentially differ from static ones by their physical and mathematical nature, and an inclusion of them into the work would destroy the integrity of description. It should be clearly realized that at the end of sixties, Russia held a far higher level relative to Science in comparison with 1925. There existed already a lot of exhaustive text-books for all areas of mechanics and scientific monographs being sometimes superior by their level to foreign analogues. The life had changed, and, in particular, the theoretical level of engineers had been essentially raised. A new kind of engineers, so called “engineers–researchers”, had appeared. Their practical results were accompanied by rather deep theoretical investigations. The whole of the above-mentioned was taken into account by A.I. Lurie when he started to work on the monograph “Theory of elasticity”. The problem was to give the exhaustive treatment of the subject, including the most important achievements of XIX–XX centuries in this field, basing on the same foundation. Naturally, the solution of this problem required the treatment of classical results in modern science language. In other words, there was a need for a thorough revision of printed matter in a huge quantity. In addition, the approaches to derivation of classical results, of course, were changed, in some cases essentially. At present, we can state that the monograph “Theory of elasticity” is in complete accordance to its designation. If anybody intends to gain the high class theoretical training in static problems of the elasticity theory, then the studying of “Theory of elasticity” is the shortest, although not too much easy, way to the aim. This is, no doubt, true with respect to the linear theory of elasticity. As for the section devoted to the nonlinear problems of the theory of elasticity, A.I. Lurie himself was not fully satisfied with the work performed[4]. A.I. Lurie's dissatisfaction with the treatment of the nonlinear problems in "Theory of elasticity" is easily explained. As it is known, the central problem in the nonlinear theory of elasticity is the formulation of the constitutive equations. In the linear theory of elasticity the constitutive equation is reduced, according to Cauchy's suggestion, to the general linear relation between the stress tensor and the strain tensor. In this case, the existence of the elastic potential is guaranteed. The only question arises when the restriction is imposed for the elastic potential to be positively defined. For an isotropic material the above restriction is reduced to the following inequalities:

$$\mu > 0, \qquad 3\lambda + 2\mu > 0, \tag{9}$$

where λ and μ are the Lamé constants. Are the restrictions (9) are necessary? The general laws of thermodynamics do not require these conditions to be true. They are not necessary from the formal mathematical point of view either. Indeed, the uniqueness and the existence of the solution for the equations of the nonlinear theory of elasticity is provided by the conditions of the strong ellipticity in statics or, which is just the same, by the conditions of the strong hyperbolicity in dynamics. The conditions lead to the necessity to fulfil the inequalities

$$\mu > 0, \qquad \lambda + 2\mu > 0, \tag{10}$$

which are more weak than the conditions (10). In the linear theory of elasticity the choice between inequalities (9) and (10) is made on the basis of the following physical principle: for any kind of the deformation of material from its natural state its internal energy, or, which is the same, its elastic potential must increase. The formulated principle was accepted in the mechanics of the elastic bodies as the stability concept. One can easily check that the material, which satisfies to the inequalities (10) with loss of the inequalities (9), can not exist continuously and it breaks spontaneously under action of arbitrary small loads. In the nonlinear theory of elasticity the situation is incomparably more complicated. Firstly, the elastic potential exists not for any relation between the stress tensor and the finite strain tensor. Therefore, it became necessary to distinguish the elastic and hyperelastic[5] materials. Secondly, the uniqueness of the solution of the static problem in the theory of elasticity is not only absent, but it must be absent. Thirdly, for the finite deformation the elastic potential does not necessarily increase along with the growth of deformation etc. In short, it is clear that the elastic potential could not be set in an arbitrary way and, at the same time, nobody knows what restrictions and why should be imposed on the elastic potential. As C. Truesdell proposed, this situation was named as a main unsolved problem of the theory of elasticity. By the time of publication of the monograph "Theory of elasticity", the problem mentioned above started to acquire the peculiar urgency. A new division of the nonlinear theory of elasticity started to form, which was named as "supplementary inequalities in the theory of elasticity". Within seventies, a variety of such inequalities was proposed, and the investigation of the consequences of violation of these inequalities started. For instance,

[4]By the way, many people told to the author of this communication that at the first acquaintance with the nonlinear theory of elasticity, they like far more the treatment of the theory in "Theory of elasticity" as compared to the much more extended monograph "Nonlinear theory of elasticity".

[5]The elastic potential exists for the latter in contrast to the former.

the problems where the condition of strong ellipticity is broken, became known as the singular problems in continuum mechanics. Certainly, A.I. Lurie could not stay aside of the questions discussed so intensively. However, in "Theory of elasticity" all these problems were not, and could not be elucidated. That is why, all at once, after the publication of "Theory of elasticity", A.I. Lurie began to work on the new monograph "Nonlinear theory of elasticity", which was published in six months after the author's decease. The process of the working on the book was rather long, since it was needed not only to prepare the material for publication, but also to carry out an enormous research work at the fore and wide front of continuum mechanics. As always, A.I. Lurie had studied in detail all the latest advances of foreign scientists. For instance, he treated the reprint (1968) of lecture notes of C. Truesdell and recommended them for translation into Russian. This translation, named as "First Course in Rational Continuum Mechanics", edited by A.I. Lurie, was published in 1975. In the course of translation A.I. Lurie was in active correspondence with C. Truesdell, who, as a result, made a lot of corrections and improvements to the initial text. Owing to this, the Russian translation of the book noticeably differed from the original. To the work on the problems described, A.I. Lurie drew his student E.L. Gurvich, with whom they published a joint paper "On the theory of wave propagation in the nonlinear elastic medium (an effective verification of the Adamar condition)", Izvestia AN SSSR, MTT, 1980, N6, pp. 110–116. The paper was of a great importance in the theoretical sense. Unfortunately, A.I. Lurie was not fated to look through the paper in the published version. As we see, up to the end of his life, despite of the disease, which took a bad course after 1976, A.I. Lurie did not cease an active scientific work. The process of creation of the monograph "Nonlinear theory of elasticity" can not be treated as anything but the heroic scientific deed. It should be mentioned that the whole book, from the first to the last line, was written with the hand of A.I. Lurie. Nevertheless, only a very thoughtful reader could discover in it the signs of the severe illness, as traces of a certain hurry. A.I. Lurie clearly realized that his days are numbered, and he feared not be able to complete the ten-year work, which was of great importance for him. In considering the book as a whole, as all monographs by A.I. Lurie, it contains a thorough treatment of the subject in terms of direct tensor calculus, which essentially facilitates the perception of the material outlined. All presently used measures of deformation and stress tensors are introduced consequently. Contrary to the linear theory, one can introduce various stress and strain tensors in the nonlinear theory, and they must be distinguished rigorously. Naturally, the main attention is paid to the theory of constitutive equations and to the formulation of restrictions for these equations, and, in particular, the restrictions for setting the elastic potential. The problems for compressible and incompressible materials are considered in detail. Note that a rubber is an important example of an incompressible nonlinear elastic material. The variational principles of the nonlinear theory of elasticity are formulated in the monograph. In particular, one can find there the principle of complementary work, proved by L.M. Zubov, the student of A.I. Lurie. This principle initiated a spirited discussion in the foreign literature. A notable attention is given to such an important area of the nonlinear theory of elasticity, as a superposition of small and finite deformations. The importance of these problems is caused by the fact that in continuum mechanics, contrary to mechanics of systems with a finite number of degrees of freedom, the only way to examine the stability of the system is to consider a superposition of small and finite deformations. The great practical importance of stability problems is undoubtful. The monograph "Theory of nonlinear elasticity" by A.I. Lurie is closed with the statement of basic facts of thermodynamics for the nonlinear elastic medium.

7 Conclusion

Even from the above and rather brief review of A.I. Lurie's works on mechanics of solids and analytical mechanics, one can see how enormous his contribution into evolution of mechanics is. Moreover, the works of A.I. Lurie on the theory of control, where he obtained some world-famed results, are not discussed in this review. As for the contribution made by A.I. Lurie into evolution of mechanics in Russia, we can not pass over the School created by him, to which hundreds of students working in various fields of mechanics belong. The students continue the life-work of A.I. Lurie. The author of the communication had an honour not only to be a student — follower of the Teacher, but to spend many hours but to spend many hours together with him in a team work at his writting table. The most striking features of A.I. Lurie were his magnificent personal qualities, his perfect honesty and scientific uprightness along with kind and responsive regards for surrounding people. You should have seen how his eyes lit up with sincere interest and curiosity when discussing new scientific results obtained by someone! At the same time, he precisely distinguished at once a true new result from the known one but "got up in new clothes".

The name of A.I. Lurie has gone down in the history of Russian mechanics for ever.