Classical and Modified Electrodynamics*

Abstract

Analysis of classical Maxwell's equations reveals following peculiarities: 1. In a general case there exists no solution for the classical system; 2. If there is a solution, it can be represented as a superposition of transversal waves propagating with light's velocity "c" and quasi-electrostatic conditions setting in instantly over the whole space.

It means, that notwithstanding the settled opinion the Maxwell's equations are not compatible with the special theory of relativity. Modified Maxwell's equations are given in this paper possessing following features:

1. There exists always a solution to them;
2. This solution is a superposition of transversal and longitudinal waves, the latter propagating with a velocity $c_1 > c$;
3. Electrostatic conditions are setting in by passing of the longitudinal wave;
4. If the scalar potential is equal to zero, solutions for classical and modified systems coincide, i.e. both systems give the same description for magnetic fields;
5. In a general case solution for the modified system transforms itself into solution for the classical system by $c_1 \to \infty$.

Classical as well as modified systems are shown to be not suitable for a correct description of interactions between the nucleus and electrons of an atom. A way to creating a new electrodynamics based on more strict principles not using quantification is shown.

1 Classical electrodynamics

Classical Maxwell's equations are described and interpreted in this section using mechanical terms. It was exactly electrodynamics, which was the source of the opinion about

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mechanistical description of Universe being basically limited and useless for investigation of electromagnetic processes. Subsequently I intend to refute this point of view.

In the modern physics Maxwell's equations are considered to be something like divine revelation, thus being just postulated. In their canonical form they can be written as follows [3, p.76]

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\varepsilon_0 c}\mathbf{j}, \tag{1}$$

where ρ represents density of charge, $\mathbf{j}$ being current's density, i.e. velocity of the charge's flow through a unit of area. The modern version is given here, which does not coincide with J.Maxwell's point of view: to Maxwell's opinion current is not necessarily connected with motion of charges. The latter circumstance will be shown to be quite significant. From (1) following condition of solvability can be obtained:

$$\nabla \cdot \mathbf{j} = -\partial\rho/\partial t. \tag{2}$$

Remark. Physicists prefer to call equation (2) a law of charge conservation considering it to be a law of Nature. From mechanical point of view generally there exist no laws of conservation, but only balance equations for certain quantities. In particular, the local charge balance equation can be written as follows:

$$\nabla \cdot \mathbf{j} = -\partial\rho/\partial t + h\,, \tag{3}$$

where h represents the volumetric speed density of the charge supplied to the given system. Even if there exist some conservation laws in Nature as a whole, they are absolutely useless for rational science, for we never examine Nature as a whole and never shall be able to do it. Mechanics and physics are investigating limited material systems being able to exchange everything, including charge, with their surroundings. Conservation laws exist for a very small class of isolated systems only. Therefore, it is in no way acceptable to consider equation (2) as a law of Nature — this is just a necessary condition of solvability for classical Maxwell's equations. It plays no such role for modified Maxwell's equations described in the next section. For them it is possible to use (3) instead of (2).

"In the course of time an opinion has formed itself on deduction of the Maxwell's equations being impossible on the basis of mechanical equations regardless of any generalizations made. Most theorists are convinced today: there is no need to deduct these equations, which are to be considered as a very successful, almost perfectly exact description of electromagnetic processes". This is a quotation from a quite old book being far from indisputable [4, pp.155-156]. Nevertheless, these words reflect quite correct the contemporary position. It suffices to take a cursory look at the system (1) to feel a doubt about its impeccability. First, there is a problem concerning treatment of the current. According to it, vector $\mathbf{j}$ is defined as speed of charge's flow through an unit of area. The system (1) is overdefined by that being insolvable in a general case. This conclusion follows just from the fact, that there are eight equations (ρ and $\mathbf{j}$ are given!) for six coordinates of the vectors $\mathbf{E}$ and $\mathbf{B}$. It is to be remarked though: the third equation of (1) follows from other three equations, if it is true for any moment of time. So in fact there are seven equations for six unknown quantities contained in (1). This contradiction can be eliminated by refusing the above treatment of the current $\mathbf{j}$. Consequences of such a refusal will be discussed in section 7. Main claims arising in connection with the system

(1) concern conclusions being obtained by mechanical interpretation of this system. Let us represent it in another, but equivalent form.

Now we introduce vector $\mathbf{u}$ satisfying following conditions:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{u}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{u}\,. \tag{4}$$

The second and the third equations of the system (1) allow introduction of such a vector. It is known, that every vector u can be represented as follows:

$$\mathbf{u} = \nabla\varphi + \nabla \times \mathbf{\Phi}\,, \qquad \nabla \cdot \mathbf{\Phi} = 0, \tag{5}$$

where potential φ is defined up to an arbitrary function of co-ordinates. It means, that addition to φ of an arbitrary function of co-ordinates does not change electric and magnetic field. By taking into account (4) and (5) it follows from the first equation of the system (1):

$$\Delta\varphi = q, \qquad \partial q/\partial t = -c\rho/\varepsilon_0, \tag{6}$$

where function q is defined up to an arbitrary function of co-ordinates. Thus, it remains to examine the fourth equation of (1). Let us represent the current $\mathbf{j}$ in following form:

$$\mathbf{j} = \nabla\varphi_* + \nabla \times \mathbf{\Phi}_*\,, \qquad \varphi_* = \frac{\varepsilon_0}{c}\frac{\partial^2\varphi}{\partial t^2}, \qquad \nabla \cdot \mathbf{\Phi}_* = 0. \tag{7}$$

Inserting these expressions into the last equation of (1), we obtain:

$$\Delta\mathbf{\Phi} - \frac{1}{c^2}\frac{\partial^2\mathbf{\Phi}}{\partial t^2} + \frac{1}{\varepsilon_0 c}\mathbf{\Phi}_* = 0. \tag{8}$$

It is easy to make certain, that the system (4)–(8) is exactly equivalent to the system (1). It allows a simple mechanical interpretation. Let us notify: according to (7) a current is not necessarily caused by motion of charges. However, in the last case the current can be treated as motion of charges too by considering electromagnetic field as consisting from two media, one of them being a continuum of negatively charged particles and the other — a continuum of positively charged particles, total density of charge being equal to zero. A current is nothing else as motion of one medium in respect to the other in this case. By such a treatment there exists no vacuum at all.

Let us now collect all the equations in one table containing two columns. In the left column there are equations of electrodynamics there, in the right one — equations of the linear dynamical theory of elasticity [5].

Electrodynamics	*Theory of elasticity*	
$\mathbf{E} = -\dfrac{1}{c}\dfrac{\partial \mathbf{u}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{u}\,,$	$\mathbf{u} = \nabla\varphi + \nabla \times \boldsymbol{\Phi}\,, \qquad \nabla \cdot \boldsymbol{\Phi} = 0$	(I)
$\mathbf{j} = \nabla\varphi_* + \nabla \times \boldsymbol{\Phi}_*\,, \quad \nabla \cdot \boldsymbol{\Phi}_* = 0$ A.	$\dfrac{1}{\mu}\,\mathbf{F} = \nabla\tilde{\varphi} + \nabla \times \tilde{\boldsymbol{\Phi}}\,, \quad \nabla \cdot \tilde{\boldsymbol{\Phi}} = 0$ B.	(II)
$\Delta\boldsymbol{\Phi} - \dfrac{1}{c^2}\dfrac{\partial^2 \boldsymbol{\Phi}}{\partial t^2} = -\dfrac{1}{\varepsilon_0 c}\,\boldsymbol{\Phi}_*$ A.	$\Delta\boldsymbol{\Phi} = \dfrac{1}{c_2^2}\dfrac{\partial^2 \boldsymbol{\Phi}}{\partial t^2} - \tilde{\boldsymbol{\Phi}}$ B.	(III)
$\Delta\varphi = q, \qquad \partial q/\partial t = -c\rho/\varepsilon_0\,,$ $\varphi_* = \dfrac{\varepsilon_0}{c}\dfrac{\partial^2 \varphi}{\partial t^2}$ A.	$\Delta\varphi - \dfrac{1}{c_1^2}\dfrac{\partial^2 \varphi}{\partial t^2} = -\left(\dfrac{c_2}{c_1}\right)^2 \tilde{\varphi}$ B.	(IV)

In this table $c_1^2 = (\lambda + 2\mu)/\rho_*$, $c_2^2 = \mu/\rho_*$, ρ_* is mass density of the medium, λ and μ are Lame's constants, $\mathbf{F}$ representing volumetric force. Velocities c_1 and c_2 define the speeds of expansion and shear waves respectively. Positiveness of deformation energy requires fulfillment of the condition $c_1^2 > 4c_2^2/3$.

I would like to remember: in contrast to electrodynamical equations theorems of solution existence are proved under sufficiently generalized assumptions for equations of the elasticity theory. Let us now interpret the equations of electrodynamics. Vector $\mathbf{u}$ in the line I is representing potential for electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. In the elasticity theory $\mathbf{u}$ is a vector of small displacements, $\mathbf{E}$ being normalized speed taken with reverse sign and $\mathbf{B}$ representing the rotor of the displacement vector being rarely used in theory of elasticity, but quite suitable for applying. Second line (II) requires no comments except of verification, that the current $\mathbf{j}$ in electrodynamics is analogous to the volumetric force in theory of elasticity. Analogy contained in line (III) will be obvious by assumptions:

$$c_2 = c\,, \qquad \tilde{\boldsymbol{\Phi}} \longleftrightarrow \frac{1}{\varepsilon_0 c}\,\boldsymbol{\Phi}_*\,.$$

Distinctions are most pronounced in the line (IV). Just the equations listed in this line define differences between electrodynamics and mechanics. In physics they are considered to be a proof of impossibility to interpret electrodynamics from mechanistical point of view, thus proving limited nature of mechanics. It would be more natural though, to admit some strangeness inherent in equations of electrodynamics and not in those of mechanics. In fact, the meaning of equation placed in the left column of the line (IV) is quite obvious. Potential φ exists for every quantity $\tilde{\varphi}$, i.e. for every volumetric force $\mathbf{F}$. Situation is different in electrodynamics. Current $\mathbf{j}$ cannot be defined arbitrarily, but is calculated (partially) from the potential φ, otherwise there may be no solution for an electrodynamical problem. This circumstance gives ground to doubts concerning "almost perfectly exact description of electromagnetic processes" with Maxwell's equations. However, considerations presented do not suffice. In contrast to the elasticity theory potential φ is not a solution of the wave equation in electrodynamics. This means electrodynamical potential φ be setting in at an instant over the whole space. In other words, Maxwell's equations lead to an infinitely high speed of signal's propagation, which contradicts scandalously to special theory of relativity (STR). Thus, STR and Maxwell's electrodynamics are not compatible with each other. There are obviously unremovable contradictions between STR and equations of the elasticity theory as well, the latter giving two values for speed of the signal's propagation. Generally, any theory giving more than one value for velocity of wave propagation cannot be compatible with the STR. To reveal more clearly analogies in equations (IV.A) and (IV.B) let us rewrite equations (IV.A) in an equivalent form:

$$\Delta\varphi - \frac{1}{c_1^2}\frac{\partial^2\varphi}{\partial t^2} = q - \frac{c}{\varepsilon_0 c_1^2}\,\varphi_*, \qquad \frac{c}{\varepsilon_0}\,\varphi_* = \frac{\partial^2\varphi}{\partial t^2}, \qquad \frac{\partial q}{\partial t} = -\frac{c}{\varepsilon_0}\,\rho\,. \tag{9}$$

First of these equations is quite analogous to equation (IV.B), provided

$$\left(\frac{c}{c_1}\right)^2 \tilde{\varphi} \;\longleftrightarrow\; \frac{c}{\varepsilon_0 c_1^2}\,\varphi_* - q\,.$$

Now it is easy to establish analogy between the volumetric force $\mathbf{F}$, the current and the charge:

$$\frac{1}{\mu}\mathbf{F} \;\longleftrightarrow\; \frac{1}{\varepsilon_0 c}\,\mathbf{j} - \left(\frac{c_1}{c}\right)^2 \nabla q$$

Assumption $\varphi_* = (\varepsilon_0/c)\,\partial^2\varphi/\partial t^2$ means a compulsory definition of a part of the volumetric force. Such an assumption appears not too convincing in mechanics as well as in electrodynamics. Nevertheless, mechanistic interpretation of the classical electrodynamics equations is obvious already, and there is no need to discuss the matter any more. Situation becomes entirely simple, if there are no charges and currents, or no volumetric forces in the elasticity theory. In this case line (IV) can be written as follows:

$$\Delta\varphi = 0 \quad \text{(IV.A)}, \qquad \Delta\varphi = \frac{1}{c_1^2}\frac{\partial^2\varphi}{\partial t^2} \quad \text{(IV.B)}$$

Equation (IV.B) transforms itself into (IV.A) by $c_1 \to \infty$. In this case the Maxwell's equations become identical to those describing oscillations of an incompressible medium, which was noted by Maxwell himself [1, p.784].

Concluding this section we want to underline: mechanical analogies for Maxwell's equations have proved themselves to be simple enough and well known to all mechanicians.

2 Modified Maxwell's equations

As noted above, classical Maxwell's equations have a grave drawback: they lead to an infinite high velocity of signal's propagation. Unfortunately, this is not the only defect of classical equations and even not the most important one, as will be shown later. Here we shall adduce a modified system of Maxwell's equations providing only finite velocities for propagation of any signals. By refusing connection described by the second equation of (9) we obtain following system:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{u}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{u}, \qquad \mathbf{u} = \nabla\varphi + \nabla \times \mathbf{\Phi}, \qquad \nabla \cdot \mathbf{\Phi} = 0; \tag{10}$$

$$\Delta\mathbf{\Phi} - \frac{1}{c^2}\frac{\partial^2 \mathbf{\Phi}}{\partial t^2} = -\frac{1}{\varepsilon_0 c}\,\mathbf{\Phi}_*; \tag{11}$$

$$\Delta\varphi - \frac{1}{c_1^2}\frac{\partial^2\varphi}{\partial t^2} = q - \frac{c}{\varepsilon_0 c_1^2}\,\varphi_*, \qquad \frac{\partial q}{\partial t} = -\frac{c}{\varepsilon_0}\,\rho, \qquad c_1^2 > 4c^2/3, \tag{12}$$

where the current is expressed by the formula:

$$\mathbf{j} = \nabla\varphi_* + \nabla \times \mathbf{\Phi}_*, \qquad \nabla \cdot \mathbf{\Phi}_* = 0. \tag{13}$$

The system (10)–(13) can be rewritten in a form more convenient for electrodynamics:

$$\nabla \cdot \mathbf{E} = \frac{\rho_*}{\varepsilon_0}, \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\varepsilon_0 c}\mathbf{j}_*, \tag{14}$$

where

$$\rho_* = \rho + \frac{1}{c_1^2}\frac{\partial}{\partial t}\left(\varphi_* - \frac{\varepsilon_0}{c}\frac{\partial^2\varphi}{\partial t^2}\right), \qquad \mathbf{j}_* = \mathbf{j} - \nabla\left(\varphi_* - \frac{\varepsilon_0}{c}\frac{\partial^2\varphi}{\partial t^2}\right). \tag{15}$$

It is necessary to add equations (12) and (13) to these relations to obtain a closed system.

System (14) appears to be like the system (1), but the meaning of it is significantly different. This difference is especially noticeable for areas, where ρ and $\mathbf{j}$ are equal to zero.

$$\rho = 0, \quad \mathbf{j} = \mathbf{0} \quad \Rightarrow \quad \varphi_* = c(t), \quad \mathbf{\Phi}_* = \mathbf{0}.$$

According to classical system (1) we shall have for this case: $\nabla \cdot \mathbf{E} = 0$. This is exactly the relation infinite velocity of signal's propagation is hidden in. Let us imagine following situation. Suppose, there existed two point charges by $t < 0$, having equal amount, but different signs and being situated at the same point by $t \le 0$. In that case $\mathbf{E} = \mathbf{0}$, $\mathbf{B} = \mathbf{0}$. for $t \le 0$. At the moment $t = 0$ these charges begin to scatter. It is easy to make certain, that potential φ will have to differ from zero by $t > 0$. By representing fields $\mathbf{E}$ and $\mathbf{B}$ with waves there would exist an area located far away from charges, where fields $\mathbf{E}$ and $\mathbf{B}$ did not come into existence yet. This area is separated from regions with existing fields $\mathbf{E}$ and $\mathbf{B}$ with a certain movable surface Σ being called wave front (see Figure 1). Let us choose a closed area bordered by the surface S. According to classical equations we shall have inside of this surface:

$$\nabla \cdot \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0.$$

These conditions are true everywhere for transversal waves, including interior of the surface S, i.e. they are true for $\mathbf{B}$ and a part of $\mathbf{E}$ represented by a transversal wave. But potential φ cannot be represented with a transversal wave, therefore it is impossible

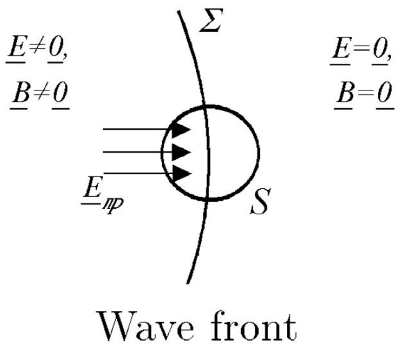

Figure 1: Wave front

for the quantity $\nabla \cdot \mathbf{E}$ to be equal to zero at the wave front, because there is something coming inside of S, but nothing comes out of it. The contradiction disappears, if we agree, that φ is not a wave and it does not have a wave front. This assumption conforms to classical electrodynamics — potential φ is setting in instantaneously over the whole space. According to modified system $\nabla \cdot \mathbf{E} \neq 0$ even if there are no charges and currents present. In the next section we shall present examples showing clearly all the points mentioned before.

Modified system (10)–(13) cannot be worse than the classical one, for the latter is contained in the first as a special case. A most "strange" feature of the system (10)–(13) is presence of waves propagating with a velocity $c_1 > c$. In the next section it will be shown, that these waves are responsible for setting in of electrostatic fields. Therefore, the modified system eliminates the abyss between electrostatics and electrodynamics inherent in classical Maxwell's equations. These equations do not allow it to infer electrostatics from a dynamical problem, electrostatics being quite a system "closed in itself". The system (10)–(13) can be considered mathematically irreproachable. How real are the waves described by equation (12)? What is the value of the velocity c_1? There are no answers to these questions yet. Pure intuition confirms existence of transversal waves (12). For myself, I doubt it in no way, for there arise unsolvable problems otherwise. Existence of waves propagating faster than light is inquestionable from experimental point of view. This fact was first established by N.A.Kosyrev [2] and then confirmed with all possible thoroughness by Academician M.M.Lavrentyev and his colleagues [6,7]. The essence of Kosyrev's experiment consists in following. He has developed a sensor to detect radiation of different types without subtilizing the nature of it. Using this sensor, Kosyrev has fixed radiation flows coming from stars. By directing the telescope at a visible star he would fix a local maximum of radiation intensity. But then he has made the most staggering discovery: he would fix a more intensive radiation by directing the telescope at the point on the sky, where the star would be really positioned at the moment of observation. Of course, there would be no star to be seen at that point, because the light coming from it will reach the Earth in distant future only. One can agree or disagree with explanations given for that by N.A.Kosyrev. But the fact of existence of radiation propagating much faster than light appears incontestable. Sure, there are no definite reasons to assume equation (12) be describing exactly that radiation, but one cannot exclude such a possibility as a matter of principle. In any case, special experiments are needed to verify the system (10)–(13) and to define the velocity c_1. It is important to note: any experimental data being explainable by classical equations would be full explained by modified equations as well.

So, the modified system cannot be worse than the classical one. Moreover, it is much better theoretically. Nevertheless, fundamental completeness of the classical as well as of the modified system appears more than doubtful. It is clear intuitively, that magnetic phenomena are being described by these systems incompletely and in a heavy distorted form, if at all. I cannot go into details here and shall confine myself to obvious remarks demonstrating fundamental incompleteness of the Maxwell's equations. To this end it is necessary to take into account facts firmly established by experimental physics.

Fact one. Interactions between the nucleus and the electrons of an atom must be of electromagnetic nature, thus they are to be described with equations of electrodynamics.

Fact two. Any atom possesses a mixed discrete-continuous spectrum to be defined experimentally.

Striving for explanation of these facts has led to establishment of quantum physics. From the point of view adopted in this paper integrity of an atom and its structure (but not the structure of the nucleus or electrons) is to be explained using equations for the second ether, i.e. equations of electrodynamics, but sure not classical electrodynamics. It is known in mechanics (see for example [8]), that mixed spectra appear by investigation of some specific problems provided presence of two main factors. First of them: presence of a boundless medium described by an operator with continuous spectrum disposed above a certain frequency (cut-off frequency). Second ether plays the role of such a boundless medium. Equations describing oscillations of an infinite beam (or a string) on an elastic bedding can be cited as an example of equations being defined on a boundless medium and having a cut-off frequency. To get a cut-off frequency from an electrodynamical equation for a boundless medium it is necessary to take into account spinorial motions being responsible for magnetic phenomena. The second factor: discrete spectrum appears below the cut-off frequency, if there are discrete particles inserted into the field of operator with continuous spectrum. Nucleus and electrons play the role of such particles. By inserting nucleus and electrons into classical or modified electromagnetic field there will appear no discrete (separated) frequencies, because the system (1) as well as the system (10)–(13) do not have any cut-off frequencies. The latter would appear in waveguides, but that is not a boundless medium anymore. Thus, electrodynamical equations are to be significantly changed to explain the structure of an atom. Exactly this is being done in quantum electrodynamics, but there are other ways remaining in the framework of the classical mechanics.

3 Electromagnetic field of a growing point charge

Some facts inherent in classical electrodynamics give rise to considerable doubts by everyone educated on traditions of classical mechanics. First of all, it is true for electrostatics, which is included in electrodynamics as a thing for itself. Every static problem in mechanics can be derived from an appropriate dynamical problem by transition to a limit. Static conditions are setting in over a body by means of certain waves. That is not so in electrodynamics: electrostatic field sets in instantaneously over the whole space. There is another fact. R.Feinmann writes [3, p.78]: "Laws of physics do not answer the question: "What will happen by a sudden appearance of a charge at a given point? Which electromagnetic effects will be observed?" There can be no answer to that, because our equations deny the very possibility of such events. If it would happen, we would need new laws, but we cannot say, what they would be like to...". It sounds very strange for a mechanician. In mechanics we suddenly apply forces of unknown sources and observe the system's reaction to these forces. Moreover, the main equations have to be solvable by arbitrarily determined external forces irrespective of the very possibility for such forces to exist. Charges and currents in electrodynamics are analogous to volumetric forces in the elasticity theory. Therefore, from mechanical point of view a satisfactory electrodynamical theory is just obliged to give a simple answer to Feinmann's question.

Suppose, there is a charge coming into existence at a given point (at the initial point of co-ordinate system). We assume this charge called a point source to be changing according to following law:

$$Q(t) = Q_0[1 - e(t)]\,, \qquad e(t) \equiv \exp\left(-2\pi\frac{t}{\tau}\right) \qquad t \geq 0$$

It is required to define disturbances of electromagnetic field connected with this source. Physicists prefer to call these disturbances just electromagnetic field as such. The problem formulated was investigated by R.Feinmann for an arbitrary function Q(t) [3, pp.145-147]. The reader can compare solution represented below with that of R.Feinmann.

The problem possesses spherical symmetry, i.e. there exist two planes of the mirror symmetry. Therefore, all quantities being represented by axial vectors, must be equal to the zero vector:

$$\mathbf{B} = \mathbf{0}\,, \qquad \mathbf{\Phi} = \mathbf{0}\,, \qquad \mathbf{\Phi}_* = \mathbf{0}\,.$$

Firstly, let us try to solve this problem using the classical system (1) under assumption of current being a motion of charges. As there are no moving charges there, $\mathbf{j} = \mathbf{0}$. We shall construct the solution using spherical co-ordinate system. In this case $\mathbf{E} = \mathcal{E}(r,t)\,\mathbf{e}_r$. Divergence $\mathbf{E}$ is equal to zero by $r \neq 0$, therefore

$$\nabla \cdot \mathbf{E} = \frac{\partial \mathcal{E}}{\partial r} + \frac{2}{r}\,\mathcal{E} = 0 \quad \Rightarrow \quad \mathcal{E}(r,t) = \frac{C(t)}{r^2}\,.$$

Using theorem of Gauss, we can define $C(t)$ and then the field $\mathbf{E}$:

$$\mathbf{E} = \frac{Q(t)}{4\pi\varepsilon_0 r^2}\,\mathbf{e}_r\,. \tag{16}$$

From the last equation of (1) we conclude, that $\partial\mathbf{E}/\partial t = \mathbf{0}$. Hence, there is no solution of the classical system (1) by $\mathbf{j} = \mathbf{0}$, because $dQ/dt \neq 0$. This is just the case, which was investigated by R.Feinmann, thus, the formula (21.13) cannot be considered to be a solution.

If we adopt a quite forced assertion about current not being necessarily connected with motion of charges and define the current as an additional unknown quantity, we shall be able to solve the classical system, because we obtain from the last equation of (1) and relation (16) for that case:

$$\mathbf{j} = -\frac{\partial \mathbf{E}}{\partial t} = -\frac{1}{4\pi r^2}\,\frac{dQ}{dt}\,\mathbf{e}_r\,.$$

Despite of existence of a formal solution it cannot be considered physically satifactory, for it sets in instantaneously over the whole space.

Let us now investigate the problem using the modified system (10)–(13). Here the current is assumed to be motion of charges, i.e. the quantity $\mathbf{j}$ is determined. We can write for that case: $\mathbf{j} = \mathbf{0} \;\Rightarrow\; \varphi_* = 0, \;\; \mathbf{\Phi}_* = \mathbf{0}\,.$

Potential $\varphi = \varphi(r, t)$ is described by equation (12)

$$\Delta\varphi = \frac{1}{c_1^2}\,\frac{\partial^2 \varphi}{\partial t^2} + q\,; \qquad \frac{\partial q}{\partial t} = -\frac{c}{\varepsilon_0}\,\rho\,.$$

Let us rewrite this equation for the function $\psi(z, t) = \partial\varphi / c\,\partial t$

$$\Delta\psi = \frac{1}{c_1^2}\,\frac{\partial^2 \psi}{\partial t^2} - \frac{1}{\varepsilon_0}\,\rho\,. \tag{17}$$

We can define electric field using following formula:

$$\mathbf{E} = -\frac{1}{c}\,\frac{\partial}{\partial t}\nabla\varphi = -\nabla\psi = -\frac{\partial\psi}{\partial r}\,\mathbf{e}_r\,.$$

Classical theorem of Gauss is no more true for this case. Suppose, the initial point of the co-ordinate system is surrounded with a small spherical volume $V_r\,,\; r \to 0\,.$ By multiplying both sides of (17) with dV_r and integrating them over volume V_r we obtain:

$$\int_{V_r} \Delta\psi\, dV_r = \frac{1}{c_1^2}\,\frac{\partial^2}{\partial t^2}\int_{V_r} \psi\, dV_r - \frac{1}{\varepsilon_0}\,Q_0\,[1 - e(t)]\,. \tag{18}$$

Using the divergence theorem, we shall have:

$$\int_{V_r} \Delta\psi\, dV_r = \int_{S_r} \mathbf{e}_r \cdot \nabla\psi\, dS_r = -\int_{S_r} \mathbf{e}_r \cdot \mathbf{E}\, dS_r\,.$$

Assuming $r \to 0$, we can write:

$$\lim_{r\to 0}\int_{S_r} \mathbf{e}_r \cdot \mathbf{E}\, dS_r = \frac{Q_0}{\varepsilon_0}\,[1 - e(t)]\,. \tag{19}$$

This relation will replace the theorem of Gauss for us.

Let us write equation (17) for an area with $r \neq 0$

$$\frac{\partial^2 r\psi}{\partial r^2} = \frac{1}{c_1^2}\,\frac{\partial^2 r\psi}{\partial t^2} \quad\Rightarrow\quad r\psi(r, t) = f(r - c_1 t)\,,$$

Here it is taken into account, that no radiation is coming from infinity. As there was no field by $t = 0$, $f(s) = 0$ by $s \geq 0\,.$ Consequently, the function $f(r - c_1 t)$ is different from zero only by negative values of the argument $s = r - c_1 t$, i.e. in the area $r < c_1 t$. Thus, we obtain a wave representation for the field $\mathbf{E}$:

$$\mathbf{E} = -\frac{\partial}{\partial r}\left[\frac{f(r - c_1 t)}{r}\right]\mathbf{e}_r = \left[\frac{f(r - c_1 t)}{r^2} - \frac{f'(r - c_1 t)}{r}\right]\mathbf{e}_r .$$

Now we can write:

$$\int_{S_r} \mathbf{e}_r \cdot \mathbf{E}\, dS_r = 4\pi\left[f(r - c_1 t) - r f'(r - c_1 t)\right].$$

By substitution this expression into (19) we get:

$$f(-c_1 t) = \frac{Q_0}{4\pi\varepsilon_0}\left[1 - e(t)\right].$$

Using this relation, we can define function f by negative values of the argument.

Finally we obtain following solution:

$$\mathbf{E}(r, t) = -\frac{Q_0}{4\pi\varepsilon_0}\,\mathbf{e}_r\,\frac{\partial}{\partial r}\begin{cases} \dfrac{1}{r}\left[1 - \exp\left(2\pi\,\dfrac{r - c_1 t}{c_1 \tau}\right)\right], & r \le c_1 t; \\ 0, & r \ge c_1 t. \end{cases} \tag{20}$$

From this expression it can be easily seen, that a quasi-static solution (16) is setting in for the area $r < c_1(t - \tau)$ by $t > \tau$. This solution is given by the classical system (1) assuming presence of non-zero current. Current is absent in the solution (20).

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